Essays on Optimal Policy without Commitment

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Dedication

I dedicate this dissertation to my parents.

Abstract

My dissertation consists of two chapters. The common theme that unifies these chapters is the determination of optimal government policy when the government cannot commit.

In the first chapter, I investigate why sovereign defaults are often accompanied by significant declines in economic aggregates, and what determines the decisions of the governments to default on their debts. I develop a model of domestic default in a production economy with financial frictions. The government finances exogenous spending with distorting taxes on labor and by issuing debt. I assume that the government cannot commit to repay its debt and characterize optimal government policies in a Markov equilibrium. A key feature of the model is that government bonds are used as collateral. Hence, defaulting on debt tightens firms' collateral constraints, thereby inducing firms to reduce their demand for labor and cut back on production. This fall in output implies defaulting on debt is costly for the government. The government trades off these costs against the distortions from taxes needed to repay the debt. Defaults occur when the costs of distorting taxes of repaying the debt outweighs the costs of output loss following default. I find that the government is more likely to default if the economy encounters a large negative TFP shock after a sequence of positive shocks. The reason is that in response to positive TFP shocks, firms increase investment and build up a high level of capital stock. If the economy is then hit with a negative shock, the collateral level is relatively high, so the cost of defaulting is lower. I calibrate the model separately to Argentine and Italian data. In the model, Argentina sustains a lower debt level with high default rate while Italy sustains a higher debt level with negligible default rate. This finding is due to the fact that the TFP process in Argentina is much more volatile, which induces its government to default more often. Furthermore, the model successfully captures the declines in output and investment associated with defaults. Output drops around 10% during defaults in the Argentine version of my model, which is close to the data; while in a counterfactual analysis, output decreases around 5% if Italy defaults.

In the second chapter, joint with Zoe Leiyu Xie, we characterize a Markov perfect equilibrium in a stochastic general equilibrium search model, where a benevolent government without commitment makes unemployment insurance policy. The policy is time consistent, as opposed to the optimal policy implemented by a Ramsey government. We contrast the Markov policy with the optimal policy. In the steady state, the Markov policy is associated with higher benefits and higher unemployment than the optimal policy. In response to a fall in productivity, the optimal policy rises on impact and then falls significantly below the steady state. In contrast, the Markov policy starts below the steady state and increases monotonically as the economy recovers. Compared to the optimal policy, the Markov policy leads to a slower recovery of unemployment. The reason behind the differences is the lack of commitment by the Markov government. The comparison highlights that with government commitment, unemployment insurance policy leads to a faster recovery of unemployment during recession. This paper thus offers a theory for why the government increases the generosity of unemployment insurance during a recession, and how such policies contribute to slow recovery in unemployment.

Contents

List of Tables

List of Figures

Chapter 1

Optimal Sovereign Defaults in the Presence of Financial Frictions

1.1 Introduction

Sovereign defaults are often associated with significant declines in economic activity in defaulting countries. For example, Levy-Yeyati and Panizza (2011), Reinhart and Rogoff (2011b), Mendoza and Yue (2012), and Tomz and Wright (2013) all find that GDP falls at least 5% below trend during default episodes. Another interesting but less well-known fact is that sovereign governments not only borrow a large fraction of their debts from domestic residents and institutions, but also default on them. For instance, Reinhart and Rogoff (2011b) document that there are many cases of domestic sovereign defaults and their macroeconomic effects are as significant as those of external defaults. They also show that domestic public debt accounts for a large fraction of total government debt in many countries. These observations raise two interesting and puzzling questions. Why are defaults often accompanied by recessions, and why do the governments default on their own citizens?

I attempt to answer these questions by developing a model of domestic default in a production economy with capital and financial frictions. The essential idea in the model is that government bonds represent a source of liquidity for the private sector by helping firms finance resources necessary to carry out production. A default disrupts this source of liquidity and weakens the economy's capacity to produce. Nevertheless, a benevolent government may still choose to default occasionally, despite its negative effects on economic activities. The reason is that a default reduces the tax burden of repaying the debt.

I focus on a closed economy to capture the idea that the holders of government bonds are domestic residents and the government still defaults on them. In the model economy, firms have to finance working capital in advance before production takes place. The amount of working capital is positively related to the scale of production. Due to limited enforcement, firms have to post collateral in order to take loans in the intraperiod loan market. Government bonds and physical capital serve as collateral for firms to obtain working capital loans. A sovereign default thus directly reduces the collateral of firms and tightens their collateral constraints, which increases financing costs. With higher financing costs, firms have to reduce their demand for labor and cut back on production. This leads to an output decline in the current period. Furthermore, this default reduces investment and output in future periods.

Despite the adverse impact, defaults do occur occasionally. In the model, the government needs to finance exogenous spending. In order to do so, the government has three instruments: raising new debt, levying taxes and/or defaulting on outstanding debt. Raising new debt has the benefit of relaxing collateral constraints. But it implies higher future taxes to repay the debt. As taxes are distortionary, the government may find it optimal to default when the level of debt is already high. The benefits of default are reduced taxes, which imply less distortion on the economy. The costs of default are reduced liquidity and decreased output. The government balances these tradeoffs, and makes default decisions when the costs of distorting taxes of repaying the debt outweighs the costs of output loss associated with default.

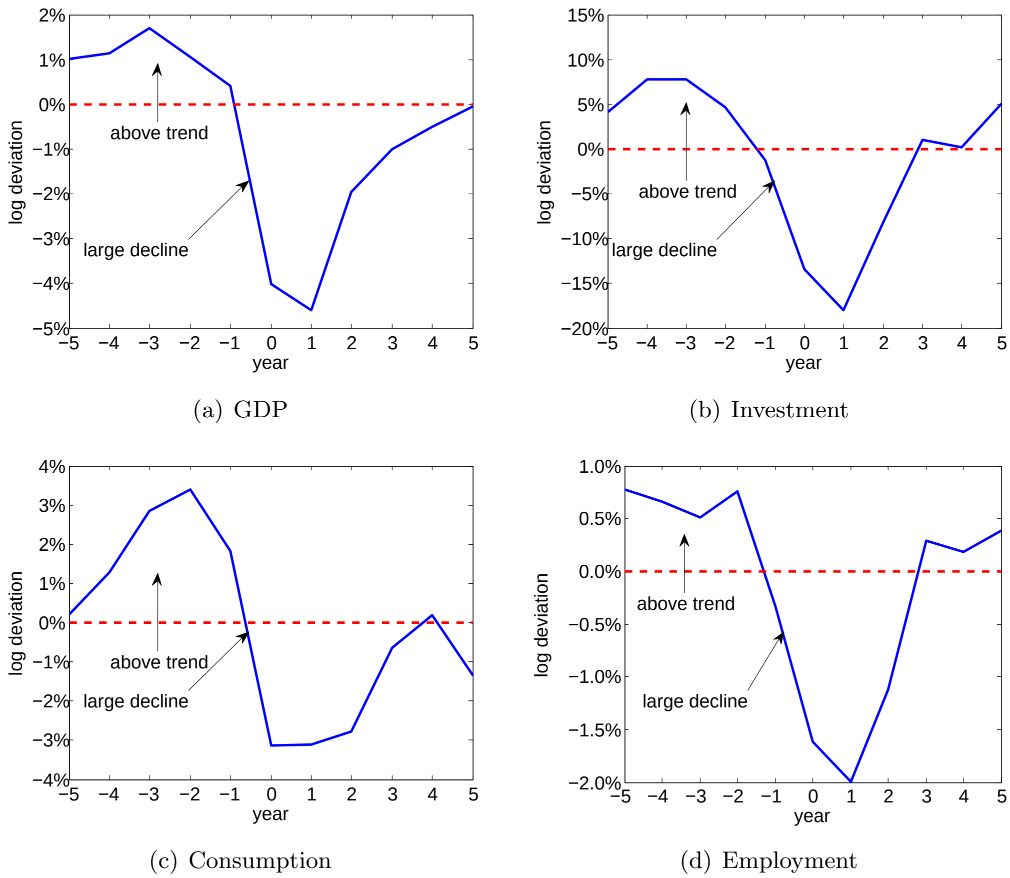
This paper therefore provides a transmission mechanism between default and production, which generates endogenous costs to sustain risky debt. Since the seminal work of Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), international macroeconomists have argued that the key to understanding sovereign debt is understanding the incentives of the government to repay its debt. That is, default must be associated with some punishment. The canonical way of modeling punishment for default is to simply assume that output decreases exogenously by a certain amount following a default. This approach is adopted by many researchers including Cole and Kehoe (2000), Aguiar and Gopinath (2006), and Arellano (2008). This paper introduces default costs that arise endogenously in the model by endowing government debt with a role of providing liquidity in private credit market and facilitating production.

By modeling the interactions between debt and production, this paper provides a new dimension to understand the government's incentives to repay or to default. In the model, the government's temptation to default is the strongest when the economy encounters a large negative TFP shock after a sequence of positive shocks. The reason is that in response to a sequence of positive TFP shocks, firms increase investment and build up a high level of capital stock. A high level of capital implies more collateral in the economy, which diminishes the importance of government bonds to be used as collateral, so the cost of defaulting is lower. At the same time, when the shock to TFP is large and negative, repaying debt is more burdensome because it requires the government to charge a higher tax rate. Hence, defaulting on debt reduces tax distortions by a greater amount relative to when the state of the economy is good. Lower default costs and higher default benefits induce the government to default when the capital stock is relatively high and TFP is relatively low. In a stationary and stochastic environment, having a sequence of positive shocks followed by a large negative shock is a low probability event, which explains why defaults occur infrequently.

Incorporating capital accumulation in the model thus leads to important implications for the data. In practice, while defaults are often characterized by decreases in output, the associated declines in investment are generally much larger. The percentage decrease in investment is usually three to four times that of the decline in GDP. In the model, the government has a strong incentive to default when the economy has a high capital stock and receives a low TFP shock. When the capital stock is high and TFP is low, the economy-wide return to capital is very low, so firms would like to reduce investment. At the same time, current consumption is low relative to future consumption, so the stochastic discount factor falls. This implies that firms discount the future at a higher rate, which further decreases investment. This reduction in investment has adverse and persistent impact on future output.

Figure 1.1 provides some evidence for the key implications of the model. Using data from 23 default events between 1980 and 2005, I plot in Figure 1.1 the log deviations of GDP, investment, consumption and employment, centered around the year of default.

[1] On the x-axis, 0 is the year of default, $-t$ and t are t years before and after respectively. Clearly, all variables decrease sharply around the time of default. While GDP falls to about 5% below trend, investment decreases to almost 20% below trend. This huge decline in investment is an under-appreciated feature in the default literature because the existing literature assumes either an endowment economy or a production economy without capital. The dynamics of investment and its interactions with default highlight the important role that capital accumulation plays both in the data and in the model.



(a) GDP

(b) Investment

(c) Consumption

(d) Employment

Figure 1.1: Macroeconomic Dynamics around Default

The inclusion of capital accumulation and the presence of collateral constraints are

[1] All variables are annual series, and HP-filtered with a smoothing parameter of 100. For descriptions of the data and the list of countries, refer to Appendix A.1.

the keys to generating the default pattern highlighted in the paper. That is, the government is more likely to default when the economy experiences a sequence of positive shocks followed by a large negative shock. This seems to be consistent with the experiences of many countries that have defaulted. Many of these countries have been growing above trend for some time before they encounter a sudden downturn and crises occur. As illustrated in Figure 1.1, GDP, investment, consumption and employment are all above trend until a year before the default crisis. This paper is able to account for this interesting pattern. Firms increase investment and build up capital stock in periods of expansion. What triggers default then is a big fall in productivity. At this time, the default costs become smaller, because the high level of capital stock built up in previous periods implies there is relatively sufficient collateral to withstand the negative shock.

In the quantitative section of the paper, I calibrate the model separately to the Argentine and the Italian economy. Argentina is an example of an emerging economy that has a history of default, while Italy is an example of a developed country that is exposed to default risk in the European debt crisis. Calibrating to Argentina provides a benchmark to test the model's performance in matching data, while calibrating to Italy allows me to answer normative questions about default risks in the ongoing debt crisis in Europe. The main result of the calibration is that the model is consistent with both types of economies.

The model generates enough government commitment to sustain observed levels of domestic debt and default frequencies in both countries. Specifically, in the simulations, Argentina sustains a low domestic debt to GDP ratio of 25% with a high annual default rate of 0.5%, while Italy sustains a high debt ratio of 59% with an almost zero default rate. [2] In the model, as in the data, Argentina defaults much more frequently than Italy, because its TFP process is much more volatile. As default is a response to alleviate debt burdens when economic conditions worsen, a very volatile TFP implies that the government will use this policy more often. With higher default rate, less debt is sustained in the Argentine economy. Furthermore, the model also successfully captures the declines in output and investment associated with defaults. In the simulations, output drops more than 10% in Argentine defaults, which is a close match to the data.

[2] The default rate here is domestic debt default rate, which is much lower than external debt default rate (a typical value is in the range of $2.5 - 3\%$) used in the literature.

In a counterfactual analysis, the model shows that if Italy defaults, output can decrease more than 5%.

My result that these two countries are able to sustain a reasonably high level of debt is consistent with the data, and also is an advance over previous quantitative default models. Empirically, the debt to GDP ratio is high in many countries. In the sample of countries depicted in Figure 1.1, the average debt/GDP ratio across all countries is about 50%. Tomz and Wright (2013) also report a cross-country average debt/GDP ratio between 40% and 60%. However, quantitative international macro and default literature have difficulty matching high debt levels. Many models either produce a debt/GDP ratio less than 10% or have to assume an extremely low discount factor. [3] In my model simulations, a debt/GDP ratio of 25% for Argentina and 59% for Italy can be sustained, with a standard discount factor and a default rate that is in line with data. This is due to fact that in the model economy, debt enhances production and resource allocation. By reducing the amount of collateral in the production sector, default leads to an instantaneous output decline. In addition, by affecting firms' investing decisions, default reduces capital accumulation, which has persistent effects.

Many empirical papers on sovereign default lend support to the idea that costs of default operate primarily through its impact on the domestic economy, rather than the external channels. Panizza, Sturzenegger, and Zettelmeyer (2009) find limited empirical support for default costs based on market exclusion and external sanction, and more support for default costs brought on the domestic economy. Arteta and Hale (2008) document that private sector experiences $30\% - 40\%$ decline in credit during sovereign debt crises and this decline is concentrated in the non-financial sector. Brutti (2011) tests the effects of default on different industry sectors, and finds that industries that are more financially dependent experience more declines in their growth during debt crisis. All these findings suggest that government bonds play a role in domestic private sector and default disrupts this function.

The theoretical framework in this paper contributes to a growing literature on the

[3] One notable exception is Chatterjee and Eyigungor (2012), who produce a high debt level by incorporating long-term debt. The reason for their sustaining a high debt level is because, with long-term debt, the government only rolls over a fraction of the outstanding debt each period. Therefore, servicing long-term debt is less onerous and the government is less sensitive to the changes in bond prices.

aggregate implications of shocks to the balance sheets of firms or financial intermediaries. The financial frictions in my model share some similarities with models studied in Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Mendoza (2010), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Jermann and Quadrini (2012). However, my model differs in two important dimensions. First, in the financial friction literature, the shocks to firm's collateral or bank's net worth often come from exogenous sources. In my paper, tightening of the collateral constraints is due to default, which is an endogenous decision of the government. Second, the financial frictions in most models manifest themselves as investment wedges. In my model, as in Jermann and Quadrini (2012), the friction is connected to the labor wedge. In accounting for fluctuations in business cycles, Chari, Kehoe, and McGrattan (2007) have shown that labor wedge plays a primary role. This feature disciplines my choice of financial friction in the paper. [4]

An important paper that introduces production into default model is Mendoza and Yue (2012). In their model, a fraction of intermediate goods are produced by foreign markets. When the government defaults, firms lose access to trade credits and those foreign intermediate goods have to be replaced by less efficient domestic intermediate goods. This source of inefficiency decreases output. While this mechanism is interesting, the empirical evidence supporting it is tenuous. Borensztein and Panizza (2009) show that exports, rather than imports, are negatively affected, and the effects are short-lived. There is also no evidence of direct trade sanctions imposed on the defaulting countries (for example, see Martinez and Sandleris (2011)).

Motivated by empirical evidence, Gennaioli, Martin, and Rossi (2014) and Sosa-Padilla (2014) study the effects of sovereign defaults on domestic banks, and the associated output losses. While they explain how default can lead to a reduction in private credit, their models, as well as Mendoza and Yue (2012)'s, do not have capital accumulation, making them unsuitable to answer questions about business cycle dynamics. In my model, firms make dynamic decisions in both physical investment (in capital)

[4] Cho and Doblas-Madrid (2013) apply the business cycle accounting methodology to a sample of 23 crises in emerging economies. They find that labor wedge plays more important role in Latin American economies while investment wedge plays more important role in East Asian economies. This also lends support to my modeling strategy as the defaulting countries are geographically concentrated in Latin America.

and financial investment (in government bonds), and capital accumulation plays an important role. Another related paper is Bocola (2014), who studies the interactions between firms, banks and the government. In his model, when the government defaults, due to the net worth effect, banks reduce their lending to firms, and output decreases. However, the dynamics of government debt and default risk are both exogenous in his model, while I endogenize both in my model.

This paper also contributes to the study of domestic sovereign defaults. In the model, domestic agents hold government bonds and the government values their welfare. Broner and Ventura (2011) argue that government debt can be sustained when a sufficiently large fraction is held by domestic residents, because the government does not default on its own citizens. However, Reinhart and Rogoff (2011b) document that there are many cases of domestic sovereign defaults. D'Erasmo and Mendoza (2013) study domestic default in an endowment economy and argue that the government defaults in order to redistribute wealth among residents. I connect default with production, and instead argue that the government defaults to reduce tax distortions.

The remainder of the paper is as follows. Section 1.2 describes the model, defines the equilibrium and characterizes the solution. Section 1.3 describes the calibration strategy. Section 1.4 shows quantitative analysis. Section 1.5 provides concluding remarks. Descriptions of data, derivations, algorithm and other materials are in Appendix A.

1.2 Model

In this section, I formulate a dynamic stochastic general equilibrium model with financial frictions in a closed economy. The modeling of financial frictions is similar to that in Jermann and Quadrini (2012). I introduce to the model economy risky government bonds. I also introduce a benevolent government who lacks commitment. The economy is populated by households, firms and the government. To finance exogenous spending, the government taxes households' labor income and issues debt, but it can default on its debt. The government chooses its tax, debt and default policies to maximize the welfare of the households. Households supply labor to firms and pay labor income tax to the government. Households also own firms and trade shares of firms. Firms operate a constant returns to scale technology that employs capital and labor. Firms hire labor from households and pays dividends to households. Firms also make investments in both physical capital and government bonds.

The key friction in this environment is that firms have to finance some working capital loan, which is related to the scale of production. Due to limited enforcement of debt contracts, firms can walk away with loans and lenders can recover only a fraction of their loans when this event occurs. This gives rise to a collateral constraint, where the firms have to pledge their collateral, comprised of both physical capital and government bonds, in order to borrow working capital. A default therefore reduces the collateral of firms and tightens their collateral constraints, forcing firms to decrease production. The presence of this endogenous cost helps to explain the sustainability of public debt and the incentives to default.

Firms in the model should not be simply taken as nonfinancial firms only. Here, I treat firms as financial firms (banks) and nonfinancial firms aggregated together. Firms' nonfinancial division makes production plans, while the financial division makes investment plans. There is no friction (cost) associated with transactions between the financial and nonfinancial divisions of firms. In Appendix A.3, I explicitly model banks and firms as separate entities. I show that the disaggregated model of banks and firms is equivalent to the baseline model in this section.

In the remainder of this section, I describe the agents in the economy, characterize the optimality conditions, describe the timing of events, and define the equilibrium.

1.2.1 Households

There is a continuum of homogeneous households. Households value consumption c_t and dislike labor n_t according to the flow utility $U(c_t, n_t)$, and they discount future at the rate $\beta \in (0,1)$. Households are the owners of firms and they trade equity shares. Households also supply labor to firms. Takings tax, price and dividend as given, a household maximizes the expected lifetime utility

$$\max_{c_t, n_t, a_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t),$$

subject to the budget constraint

$$c_t + p_t a_{t+1} = (1-\tau_t) w_t n_t + (p_t + d_t) a_t,$$

where w_t is the wage rate, τ_t is the tax rate on labor income, a_t is the equity shares, d_t is the dividend received from owning shares, and p_t is the market price of shares. The intratemporal and intertemporal Euler equations are

$$\frac{U_n(c_t, n_t)}{U_c(c_t, n_t)} = -(1 - \tau_t)w_t, \tag{1.1}$$

$$p_t U_c(c_t, n_t) = \beta \mathbb{E}\left[(d_{t+1} + p_{t+1})U_c(c_{t+1}, n_{t+1})\right]. \tag{1.2}$$

The first condition determines the supply of labor, and the second condition determines the purchase of equity shares.

1.2.2 Firms

There is a continuum of firms of measure 1. Firms operate a constant returns to scale production technology, $F(z_t, k_t, n_t) = z_t k_t^{\alpha} n_t^{1-\alpha}$. The variable z_t is the stochastic technology shock common to all firms, k_t is the stock of capital, and n_t is the labor input. Labor is hired at competitive labor market each period. Capital k_t is chosen at time $t-1$ and thus predetermined at time t. Capital evolves according to $k_{t+1} = (1-\delta)k_t + i_t$, where δ is the depreciation rate and i_t is investment.

Besides investing in physical capital, firms also invest in government bonds. Government bond is a risky one-period security, which pays one unit of consumption in the following period if the government repays and $1 - \lambda$ unit of consumption if the government defaults. λ is the haircut of the debt should the government defaults.

Within the period, firms need to raise funds with an intraperiod loan, ℓ_t, to finance working capital. Working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of output. The intraperiod loan is repaid at the end of the period, and there is no interest.

A firm starts off the period with capital k_t and government bond b_t. For the convenience of illustration, I assume that the government chooses not to default for the time being. Before producing, the firm needs to choose labor n_t, dividend d_t, investment $i_t = k_{t+1} - (1 - \delta)k_t$, and new government bond b_{t+1}. Since these allocations are made before the realization of output, the intraperiod loan raised by the firm is

$$\ell_t = w_t n_t + d_t + i_t + q_t b_{t+1} - b_t \tag{1.3}$$

where w_t is the wage rate and q_t is the price of government bond. The firm's budget constraint is

$$d_t + k_{t+1} + q_t b_{t+1} = (1-\delta)k_t + F(z_t, k_t, n_t) - w_t n_t + b_t. \tag{1.4}$$

It is straightforward to verify that the intraperiod loan is equal to the firm's output, i.e, $\ell_t = F(z_t, k_t, n_t)$. [5]

In the model economy, there is limited enforcement of debt contracts. Firms have incentives to default on their obligations after the realization of output but before repaying the intraperiod loan. In case of default, lenders will try to liquidate firm's assets, which are physical capital k_{t+1} and government bond $q_t b_{t+1}$. Because government bond is priced at q_t in period t, the value of bond is worth $q_t b_{t+1}$. Suppose that lenders can recover a fraction ξ of the firm's net worth $k_{t+1} + q_t b_{t+1}$. [6] In Appendix A.2, there is a description of the renegotiation process between the firm and the lender in the event of default. Based on the outcomes of the renegotiation, we can derive the firm's collateral constraint,

$$\xi(k_{t+1} + q_t b_{t+1}) \geq \ell_t = F(z_t, k_t, n_t).$$

The amount of intraperiod loan cannot be higher than a fraction ξ of firm's assets $k_{t+1} + q_t b_{t+1}$. The capacity to borrow is bounded due to the limited enforceability of debt contracts. On the one hand, higher working capital loan makes the collateral constraint tighter. On the other hand, higher capital stock and/or higher holding of government bonds relax the collateral constraint. [7]

If the government chooses to default, then the firm's budget constraint and collateral constraint are

$$\begin{aligned} d_t + k_{t+1} + q_t b_{t+1} &= (1-\delta)k_t + F(z_t, k_t, n_t) - w_t n_t + (1-\lambda)b_t \\ \xi(k_{t+1} + q_t b_{t+1}) &\geq \ell_t = F(z_t, k_t, n_t) \\ b_{t+1} &= (1-\lambda)b_t \end{aligned}$$

[5] Substituting the intraperiod loan equation (1.3) into the firm's budget constraint (1.4) gives $\ell_t = F(z_t, k_t, n_t)$.

[6] An equivalent way is to assume that with probability ξ, the lenders can recover the full value $k_{t+1} + q_t b_{t+1}$. With probability $1-\xi$, the recovery value is zero.

[7] Dang, Gorton, and Holmstrom (2012) provide some micro foundation and explanation that government debt is an optimal form of collateral and liquidity.

where λ is the haircut on the outstanding debt. I assume that the government cannot raise new debt in the period in which it defaults. Therefore the fraction of debt that is not defaulted is carried over to the next period. It can be priced because agents can still buy and sell these debts in the secondary market. [8]

Recursive Formulation of the Firm's Problem

The individual states are the capital stock k and government bond b. The aggregate states, to be specified later, are denoted by S. Given the individual and aggregate states, let $W(k, b; S)$ be the value of the firm. The firm's problem is

$$\begin{aligned} W(k,b;S) &= \max_{d,n,k',b'} d + \beta \mathbb{E}\left[\frac{U_c(c',n')}{U_c(c,n)} W(k',b';S')\right] \qquad (1.5)\\ &\text{subject to} \\ d + k' + qb' &= (1-\delta)k + F(z,k,n) - wn + (1-D\lambda)b, \qquad (1.6)\\ \xi(k'+qb') &\geq F(z,k,n). \qquad (1.7)\end{aligned}$$

where $D \in \{0,1\}$ is an indicator function for whether the government repays or defaults on its debt. Since firms are owned by households, $\beta\frac{U_c(c',n')}{U_c(c,n)}$ is firm's stochastic discount factor. An individual firm takes as given the stochastic discount factor, wage rate and price of bond, which are determined in the general equilibrium.

As mentioned earlier, firms here should be taken as financial firms (banks) and nonfinancial firms aggregated together. In Appendix A.3, I describe a disaggregated model of banks and firms. I show that the baseline model and the disaggregated model are equivalent.

Let μ be the Lagrange multiplier associated with the collateral constraint (1.7), the first-order conditions for n, k', and b' are [9]

$$\begin{aligned} F_n(z,k,n) &= \frac{w}{1-\mu}, \qquad (1.8)\\ 1-\xi\mu &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-\delta+(1-\mu')F_k(z',k',n')\right]\right), \qquad (1.9)\\ (1-\xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-D'\lambda\right]\right). \qquad (1.10)\end{aligned}$$

[8] Broner, Martin, and Ventura (2010) show that sovereign debt can be sustained without any default penalties due to the presence of secondary markets.

[9] The detailed derivation is provided in Appendix A.4.

Equation (1.8) is the optimality condition for labor, where the marginal productivity of labor, $F_n(z,k,n)$ is equalized to the marginal cost. The marginal cost is the wage rate w augmented by a wedge that depends on the tightness of the collateral constraint, $\frac{1}{1-\mu}$. If the collateral constraint is not binding, i.e., $\mu = 0$, then the optimality condition is reduced to the standard condition in the macro literature. On the other hand, a tighter constraint (a higher μ) increases the effective cost of labor and reduces its demand. A channel through which default risks are transmitted to the real sector of the economy is through the demand for labor.

To see how default affects the production decisions of firms, I rewrite the collateral constraint. Using the budget constraint (1.6) to eliminate $k' + qb'$, the collateral constraint (1.7) can be rewritten as

$$\frac{\xi}{1-\xi}\left[(1-\delta)k + (1-D\lambda)b - wn - d\right] \geq F(z,k,n).$$

At the beginning of the period, k and b are given. Suppose the collateral constraint is binding and the government defaults. Then the firm has to reduce dividend d and/or cut employment n. However, even without direct dividend adjustment cost, there is an indirect cost to adjust dividend. From the budget constraint, a reduction in dividend implies that the firm has to increase its investment by about the same amount. As will be shown later, the government tends to default when the TFP is low. Increasing investment when the TFP is low further depresses the return to capital. In other words, if the firm wants to lower the cost of reduced production, it has to increase the cost of decreased return to investment. Thus there is a limit to the extent to which investment (dividend) can be increased (reduced). Therefore firms use a combination of reducing dividend and reducing employment in response to default.

1.2.3 The Government

The government is a benevolent social planner, using its policy instruments to maximize the welfare of the representative household. However, it has no commitment to its debt and tax policies. Every period, the government decides if it is going to default on its debt, how much to tax τ, and how much new debt B' to issue if it chooses not to default.

Tax is on labor income while debt is one-period short-term securities. [10] In addition, the government has to finance some public spending. I assume public spending as a fraction of GDP is constant at a ratio of g. [11] The government finances its spending and pays back its outstanding debt by levying taxes on household's labor income and by issuing new government bonds to firms.

Government bonds are risky because the government can default $D \in \{0, 1\}$ in every period, where $D = 0$ means repayment and $D = 1$ means default. When the government defaults, it writes off a fraction $\lambda \in [0, 1]$ of its outstanding debt. The parameter λ can be seen as the "haircut" that the government imposes on bondholders when it defaults. Let B be the amount of the outstanding debt. If the government chooses to repay its debt, it can issue new bond B' at price q. If the government chooses to default on its debt, it cannot issue new debt for the current period and roll over the non-defaulted portion of the debt. This means that the non-defaulted debt can still be traded. Denoting by τ the tax rate on labor income and Y the aggregate output in the economy, the government's budget constraint is

$$\begin{cases} gY + B = qB' + \tau wn & \text{if repays,} \\ gY + (1-\lambda)B = q(1-\lambda)B + \tau wn & \text{if defaults.} \end{cases}$$

In the sovereign default literature, there are two common assumptions. The first is that the government defaults on all of its debt. However, partial defaults are prevalent phenomenon in reality. For example, Sturzenegger and Zettelmeyer (2008) find that there is a wide variation in haircuts, from 13% to 73%. Cruces and Trebesch (2013) find similar pattern, with an average haircut of 37%. Benjamin and Wright (2013), and Arellano, Mateos-Planas, and Rios-Rull (2013) both find an average haircut around 50%. These facts guide my assumption of partial default in the model, where λ is the haircut on debt.

Another common assumption in the literature is that the government, if defaults, is excluded from the credit market for an exogenous period of time. However, a vast

[10] Due to the already complicated computation of the model, I abstract from capital income tax and long-term debt, even though both can generate interesting dynamics. For interactions between optimal labor and capital income taxes, see Chari and Kehoe (1999) and Conesa, Kitao, and Krueger (2009). For risky long-term debt, see Chatterjee and Eyigungor (2012) and Arellano and Ramanarayanan (2012).

[11] Public spending g can be endogenized by assuming a utility function that values public spending. It can also be made stochastic at the cost of adding one more state variable.

empirical work has shown that governments in defaults get back to the credit market fairly quickly. Arellano, Mateos-Planas, and Rios-Rull (2013) document that countries continue to have access to credit markets during defaults. Gelos, Sahay, and Sandleris (2011) report that credit market exclusion is short-lived and has declined to two years since the 1990s. I therefore include limited market exclusion, where the government cannot issue debt only in the period in which it defaults. Since the time period is annual in the model, it means the credit market exclusion lasts for one year.

1.2.4 Timing

The timing of events is illustrated in Figure 1.2 and it proceed as follows:

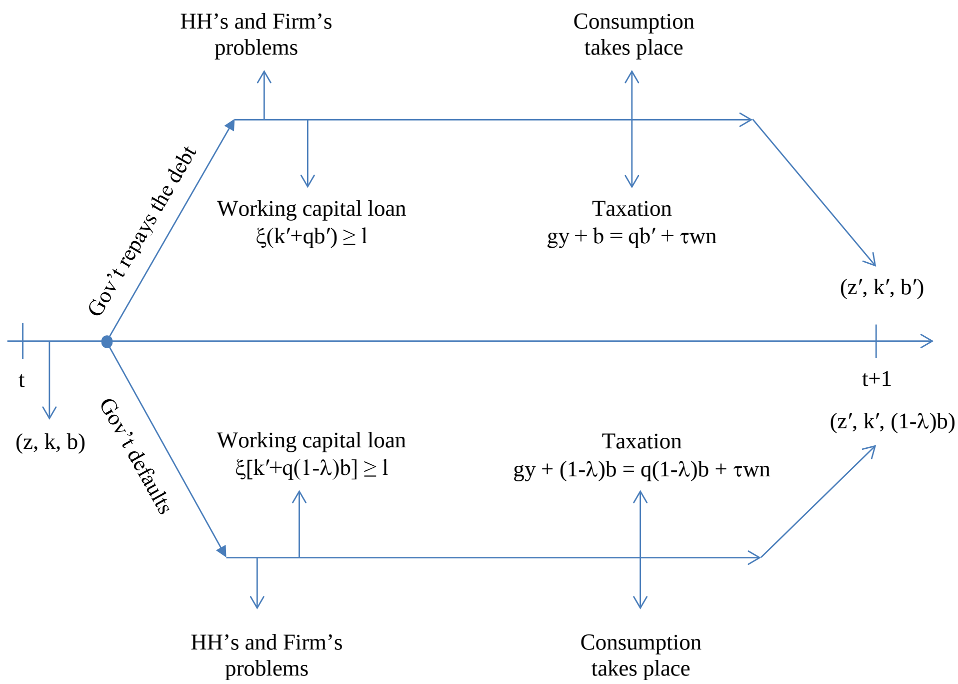


Figure 1.2: Timing

1. At the start of period t, the exogenous productivity shock z_t is observed.

2. The aggregate state variables are TFP, capital and bond (z_t, k_t, b_t).

3. The government makes default decision: $D_t \in \{0, 1\}$.

- If the government chooses to repay ($D_t = 0$), the following happens:
  - The government pays back its outstanding debt, issues new debt, and announces tax plan.
- If the government chooses to default ($D_t = 1$), the following happens:
  - The government writes off a fraction λ of its outstanding debt and announces tax plan.
  - The government cannot issue new debt in this period and rolls over the non-defaulted debt.

4. Firms decide how much bond to purchase, labor to hire, investment to make and dividend to issue. Firms borrow to meet the working capital requirement.

5. Households decide how much labor to supply, how much equity shares to purchase and how much to consume.

6. All markets clear. Firms repay working capital loans. Households pay taxes and consume.

7. Period t ends. The new level of capital stock is k_{t+1}. The new level of government debt is b_{t+1} if the government repays or $(1-\lambda)b_t$ if the government defaults.

1.2.5 Recursive Equilibrium

In most of the models of sovereign default, there are no individual agents and the government is the only agent that performs optimization problem. While in many models in the financial friction literature, there are either no government or government policies are exogenous. In contrast, this paper features both individual agents and the government as optimizing agents. Individuals take as given expected government policies to make their intratemporal and intertemporal optimizations. The government takes as given individual policies to maximize the welfare of the residents. Given this structure and the time inconsistency problem, I focus on Markov perfect equilibrium in which policies are functions of payoff relevant state variables. In particular, I define the equilibrium in two steps. First, I define a recursive competitive equilibrium given a government policy. Then I define a Markov perfect equilibrium as the competitive equilibrium associated with the optimal government policy.

Denote the aggregate states by $s = \{z, K, B\}$, where z the productivity shock, K is the aggregate capital stock and B is the level of government debt. Denote government polices by $\pi(s) = \{D(s), B'(s), \tau(s)\}$. Let the superscript f stand for firms.

Definition 1. (Recursive Competitive Equilibrium) Given government policies $\pi(s)$, a recursive competitive equilibrium is defined as a set of functions for (1) household's consumption $c(s, \pi(s))$, labor supply $n(s, \pi(s))$, and equity share $a'(s, \pi(s))$; (2) firm's dividend $d(k, b; s, \pi(s))$, labor demand $n^f(k, b; s, \pi(s))$, capital $k'(k, b; s, \pi(s))$ and bond $b'(k, b; s, \pi(s))$; (3) wage $w(s, \pi(s))$, price of bond $q(s, \pi(s))$, and price of equity $p(s, \pi(s))$; (4) law of motion for aggregate states $s' = \Psi(s, \pi(s))$ such that: [12]

1. household's policies satisfy household's optimality conditions (1.1)-(1.2);

2. firms' policies solve firm's problem (1.5);

3. $w(s, \pi(s))$ clears labor market: $n(s, \pi(s)) = n^f(K, B; s, \pi(s))$;

4. $q(s, \pi(s))$ clears bond market: $b'(K, B; s, \pi(s)) = B'(s)$;

5. $p(s, \pi(s))$ clears equity market: $a'(s, \pi(s)) = 1$;

6. aggregate resource constraint holds:

$$c(s, \pi(s)) + K'(s, \pi(s)) = (1 - \delta)K + (1 - g)zK^{\alpha}n(s, \pi(s))^{1-\alpha};$$

7. the law of motion $\Psi(s, \pi(s))$ is consistent with individual decisions:

$$K'(s, \pi(s)) = k'(K, B; s, \pi(s)).$$

I now proceed to define the government's problem. In each period, the government chooses tax, debt and default policies to maximize the welfare of households, while subject to two conditions. Government budget constraint has to be satisfied, and the resulting allocations have to constitute a competitive equilibrium as defined above.

[12] Technically, it should be $x(s)$ instead of $x(s, \pi(s))$, where x is the variable of interest. I explicitly write in this form to show the dependence of allocations and prices on government policies.

Since the government does not have commitment to future policy rules, it chooses its policies in a period taking as given future governments' policy rules. A Markov perfect equilibrium is characterized as the fixed point in these policies rules, whereby the current government does not have incentives to deviate from future governments' policy rules, thus making these rules time-consistent.

From now on, with an abuse of notation, I will denote the aggregate capital and debt by lower-case letters as long as they do not cause confusion. At the beginning of every period, the government decides if it is going to default,

$$\begin{aligned} V(z,k,b) \quad &= \quad \max\left\{V^r(z,k,b), V^d(z,k,b)\right\} \\ D=0 \quad \text{if} \quad &\quad V^r(z,k,b) \geq V^d(z,k,b) \\ D=1 \quad \text{if} \quad &\quad V^r(z,k,b) < V^d(z,k,b) \end{aligned}$$

where V^r is the value of repaying and V^d is the value of defaulting. If the government repays, it solves the following problem:

$$\begin{aligned} V^r(z,k,b) &= \max_{c,n,d,k',b',\tau,w,q,\mu} U(c,n) + \beta\mathbb{E}\left[V(z',k',b')\right] \\ \text{subject to} & \\ c &= (1-\tau)wn + d \\ d + k' + qb' &= (1-\delta)k + zk^{\alpha}n^{1-\alpha} - wn + b \\ gzk^{\alpha}n^{1-\alpha} + b &= qb' + \tau wn \\ \frac{U_n}{U_c} &= -(1-\tau)w \\ (1-\alpha)zk^{\alpha}n^{-\alpha} &= \frac{w}{1-\mu} \\ (1-\xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1-D(s')\lambda\right]\right) \\ 1-\xi\mu &= \beta\mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1-\delta+(1-\mu(s'))\alpha z'k'^{\alpha-1}n(s')^{1-\alpha}\right]\right) \\ \xi(k'+qb') &\geq zk^{\alpha}n^{1-\alpha}, \mu \geq 0, \text{ and } \mu[\xi(k'+qb') - zk^{\alpha}n^{1-\alpha}] = 0 \end{aligned}$$

where $s' = (z',k',b')$, and the government takes as given future government's policy rules and future competitive equilibrium allocation rules. The competitive equilibrium conditions are subsumed into the value function as constraints to the government's problem. [13] Similarly, if the government defaults, it solves the following problem:

$$
\begin{aligned}
V^d(z,k,b) &= \max_{c,n,d,k',\tau,w,q,\mu} U(c,n) + \beta\mathbb{E}\left[V(z',k',(1-\lambda)b)\right] \\
\text{subject to} & \\
c &= (1-\tau)wn + d \\
d + k' + q(1-\lambda)b &= (1-\delta)k + zk^\alpha n^{1-\alpha} - wn + (1-\lambda)b \\
gzk^\alpha n^{1-\alpha} + (1-\lambda)b &= q(1-\lambda)b + \tau wn \\
\frac{U_n}{U_c} &= -(1-\tau)w \\
(1-\alpha)zk^\alpha n^{-\alpha} &= \frac{w}{1-\mu} \\
(1-\xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1 - D(s')\lambda\right]\right) \\
1-\xi\mu &= \beta\mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1-\delta+(1-\mu(s'))\alpha z'k'^{\alpha-1}n(s')^{1-\alpha}\right]\right) \\
\xi(k'+q(1-\lambda)b) &\geq zk^\alpha n^{1-\alpha}, \mu \geq 0, \text{ and } \mu[\xi(k'+q(1-\lambda)b) - zk^\alpha n^{1-\alpha}] = 0
\end{aligned}
$$

where $s' = (z', k', (1-\lambda)b)$. For both sets of value functions, the first constraint is household's budget constraint, the second is firm's budget constraint, the third is the government budget constraint, the fourth is household's intratemporal Euler equation, the fifth is firm's demand for labor, the sixth and seventh are the firm's intertemporal Euler equations for bond and capital respectively, the last constraints are the collateral constraint and the complementary slackness conditions (Kuhn-Tucker conditions). Now we are ready to define the Markov perfect equilibrium.

Definition 2. (Markov Perfect Equilibrium) A stationary Markov perfect equilibrium is defined as a set of functions for (1) value functions $\{V(s), V^r(s), V^d(s)\}$; (2) policy functions $\{D(s), \tau(s), b'(s)\}$; (3) allocation rules $\{c(s), n(s), d(s), k'(s)\}$; (4) pricing functions $\{w(s), q(s)\}$ such that:

1. given pricing functions, allocation rules and future government policy functions, current government policy functions $\{D(s), \tau(s), b'(s)\}$ solve the government's maximization problems $\{V(s), V^{nd}(s), V^d(s)\}$;

[13] This is similar to the primal approach in the Ramsey problem. It is as if the government chooses all allocations in the economy. But the presence of these constraints guarantees that these allocations are the competitive equilibrium allocations.

2. given pricing functions and government policy functions, $\{c(s), n(s), d(s), k'(s)\}$ are the competitive equilibrium allocation rules;

3. policy functions obtained by solving the government problems coincide with future government policy functions that the government problems take as given;

4. markets clear.

1.2.6 Tradeoffs in Government Policies

In this subsection, I discuss the tradeoffs of the government's policies. When choosing its policies on tax, debt and default, the government weighs the benefits and costs. I first start with the equilibrium condition for labor, by combining the labor supply equation (1.1) and labor demand equation (1.8) into

$$-\frac{U_n(c,n)}{U_c(c,n)} = (1-\tau)(1-\mu)F_n(z,k,n).$$

$-\frac{U_n}{U_c}$ is the marginal rate of substitution while F_n is the marginal product of labor. Following the tradition in the business cycle accounting literature, I refer to $(1-\tau)(1-\mu)$ as the labor wedge, which in this case is a combination of tax distortion (τ) and liquidity (μ is the tightness of the collateral constraint).

First, we consider increasing the amount of debt. Issuing more debt (higher b') reduces current tax rate (lower τ) and relaxes the collateral constraint (lower μ). However, higher b' implies a higher tax rate next period. Thus, it faces this intertemporal tradeoff of decreasing current distortion at the expense of increasing future distortion. Furthermore, there is a limit to which current distortion can be reduced. Because the government has no commitment and can default on its debt, higher debt implies higher default risk and lower bond price q. Thus, when the debt level is high enough, increasing debt may even increase current tax and tighten the collateral constraint. Similarly, reducing the debt level has the opposite effects.

Second, we consider defaulting on debt. As the government does not need to pay back a fraction of its debt, it has the benefit of reducing current tax rate (lower τ). But it undermines the firm's ability to raise working capital by destroying a financing instrument and tightening the collateral constraint. Reducing the tax rate increases labor supply of households while tightening the collateral constraint decreases labor demand of firms. Default has this intratemporal tradeoff. Furthermore, default has another benefit of starting off with a lower level of debt in the next period. As the model features a closed economy, it does not include the conventional tradeoff where default represents a net transfer of resources from foreigners.

Next, we consider the government policies' impact on investment and capital. Due to arbitrage, the expected returns on capital and bonds have to be equalized. On the negative side, increasing debt crowds out capital, as more resources are diverted away from investment in capital to the purchase of government bonds. On the positive side, bond provides an additional means of financing working capital loans, therefore effectively loosens the collateral constraint. Without government bonds, firms can increase capital to relax the collateral constraint, but it depresses the return to capital. To formalize this, we look at the firm's Euler equation for capital (equation (1.9), reproduced below for convenience),

$$1 - \xi\mu = \beta\mathbb{E}\left(\frac{U_{c'}}{U_c}\left[1 - \delta + (1 - \mu')F_{k'}\right]\right).$$

The expected return to capital is $\frac{1-\xi\mu}{\beta}$, which is lower than the rate of time preference when the collateral constraint is binding. Hence, increasing bond increases the amount of collateral and increases the expected return to capital. Although there is no explicit tax on capital, μ' is effectively a tax rate on capital. As higher amount of bonds means the future collateral constraint will be less binding (small μ'), increasing debt effectively reduces future capital income tax. However, as default makes the collateral constraint more binding, it decreases the return to capital and increases effective capital tax. Therefore, in the model, sovereign default shifts the tax burden from labor to capital, so default can also be viewed as a tradeoff between labor wedge and investment wedge.

1.3 Calibration

In this section I describe the calibration strategy. In particular, I first describe the data and the calibration procedure, and then comment on the solution method. I calibrate the model to the Argentine economy for the period of 1980-2010. Argentina is a typical emerging economy studied in the literature because it has defaulted several times in history. I also do robustness check for the model by calibrating it to the Italian economy. Since the procedures are similar for the two calibrations, I show them for only the Argentine economy. [14]

1.3.1 Data and Calibration

I collect annual data on national accounts, employments and public debt for Argentina from 1980-2010. Data on GDP, households' consumption and investment are from the World Development Indicators by the World Bank. Data on government expenditure is from the Latin American and Caribbean Macro Database by the Inter-American Development Bank (IADB). Data on employments is from the Conference Board Total Economy Database. IADB Database and Panizza (2008) both provide public debt data and a breakdown into domestic and external public debt. I merge their data and find that Argentina has an average domestic debt to GDP ratio of 25%. [15]

The model is calibrated to an annual frequency. All the parameter values are listed in Table 1.1. Household's discount factor β is set to 0.95, a common value in the real business cycle literature with annual frequency. Household's utility function takes the form of

$$U(c,n) = \log(c) - \chi\frac{n^{1+\nu}}{1+\nu}$$

which is consistent with the preferences used in the growth and real business cycle literature. As the data on worked hours is not available for Argentina, I assume the steady state hours equal to 0.32, a standard value in the literature. [16] To have hours equal to 0.32, the parameter for disutility of labor χ is calibrated to be 4.35. The curvature of labor supply ν is chosen to match the Frisch elasticity of labor supply, $1/\nu$. The estimates for this elasticity vary considerably in the literature. I choose ν to be 0.5, implying a Frisch elasticity of 2, within the range of estimates in the macro literature.

In the Cobb-Douglas production function, the capital share α is set to 0.3, a standard capital share in GDP. The capital depreciation rate δ is chosen to be 0.09 to match an average investment/GDP ratio of 0.19 over the period 1980-2010. The government spending to GDP ratio g is set to 0.22, reflecting an average ratio of General Government

[14] Calibration results for Italy are in Section 1.4.4.
[15] The average total public debt to GDP ratio in Argentina is 50%.
[16] The worked hours is 0.32 and the capital share is 0.3 in the Italian data. I assume the same numbers for Argentina so as to make comparisons easier.

Table 1.1: Parameters for Argentina

Calibrated Parameters		Value	Target Statistics
Household's discount factor	β	0.95	Standard value
Curvature of labor supply	ν	0.5	Frisch elasticity = 2
Disutility of labor	χ	4.35	Steady state hours = 0.32
Capital share in output	α	0.3	Standard value
Capital depreciation rate	δ	0.09	Investment/GDP = 19%
Gov't spending/GDP	g	0.22	Gov't spending/GDP = 22%
Collateral parameter	ξ	0.416	Mean debt/GDP = 25%
Partial default	λ	0.5	Haircut = 50%
Autocorr. of prod. shock	ρ_z	0.82	Autocorr. of TFP = 0.82
Std. dev. of prod. shock	σ_z	0.044	Std. dev. of TFP = 0.044

Expenditures to GDP of 22% in Argentina. For the partial default parameter λ, the empirical counterpart is the haircut associated with defaults. Benjamin and Wright (2013), and Arellano, Mateos-Planas, and Rios-Rull (2013) both find an average haircut around 50%. Therefore I set λ equal to 0.5 to match the average haircut. This number is also within the range of 13% to 73%, estimated by Sturzenegger and Zettelmeyer (2008).

The collateral parameter ξ studied in this paper is fairly abstract and has no direct empirical counterpart. For this reason, I follow the tradition in the financial friction literature and use the model's implications to relate the tightness of the collateral constraint to a set of observable variables. Recall that the collateral constraint is $\xi(k' + qb') \geq F(z, k, n) = y$. Suppose the constraint is binding at the deterministic steady state, I can rewrite it as

$$\xi \left(\frac{k}{y} + q\frac{b}{y} \right) = 1$$

I first guess the parameter ξ and solve the model. It generates a value for $\frac{b}{y}$ at the steady state. I then match this value of $\frac{b}{y}$ to the debt/GDP ratio in the data and verify the collateral constraint is indeed binding at the steady state. The average of domestic public debt to GDP ratio in Argentina is 0.25. The required value is $\xi = 0.416$.

To calibrate the TFP parameters ρ_z and σ_z, I follow the standard Solow residuals approach. I use annual data on GDP, investment, and labor from 1960 to 2010 in Argentina. [17] I then apply the perpetual inventory method to construct the capital stock. Using the Cobb-Douglas production function, I derive

$$\log(z_t) = \log(y_t) - \alpha \log(k_t) - (1 - \alpha) \log(n_t).$$

Given the value of α and the empirical series of y, k,and n, I construct the series for Solow residuals. I then linearly detrend the Solow residuals. With an abuse of notation, I rename the detrended TFP as $\log(z_t)$ and assume it follows the following process

$$\log(z_{t+1}) = \rho_z log(z_t) + \epsilon_{z,t}$$

where $\epsilon_{z,t}$ is i.i.d. with standard deviation σ_z. I estimate the equation and get $\rho_z = 0.82$ and $\sigma_z = 0.044$. I then use Tauchen (1986) quadrature method to construct a Markov approximation to this process with 9 realizations.

1.3.2 Solution Method

As analytical solutions are not available for this model, I use numerical methods to analyze the model's behavior. The model is solved using a global solution that relies on projection methods. In particular, I use policy function iteration to solve for competitive equilibrium and value function iteration to solve for the government problem. As the collateral constraint is not always binding, I also need to check for occasionally binding constraints. [18]

Given the current government policies, expected government policies and agent's conditional expectations, and assume the collateral constraint is binding, I solve for the competitive equilibrium allocations and prices by solving a system of equations using a nonlinear optimization routine at every grid point. If the collateral constraint turns out to be not binding, I set the multiplier to be zero and solve the system of equations again. Given the set of competitive equilibria and expected government policies, I solve for the optimal current government policies as the policies that maximize the

[17] I use the longer time series here to construct the capital stock and TFP process.

[18] Christiano and Fisher (2000) provide an algorithm for solving models with occasionally binding constraints.

value functions. I iterate until the value functions, government policies and agent's expectations all converge. This means that neither the government nor the agents have incentives to deviate from their policies. Interpolation method is used for points off the grid. Appendix A.5 provides a detailed description of the algorithm.

1.4 Quantitative Analysis

In this section I focus on the quantitative analysis of the model. As mentioned earlier, I apply the model to both the Argentine and the Italian economies. I consider this exercise is interesting for three reasons. First, the ongoing European debt crisis shows that even developed countries are not immune from default risk, but little work has been done on default risk in developed economies. For this reason, I pick Italy because it is an important developed country that is exposed to default risk and it has a large domestic debt to GDP ratio of 60%. Second, by comparing an emerging and a developed country, I show that the model is consistent with both types of economies. Third, even though Italy has not defaulted, I attempt to answer question on what would happen to its economy if Italy defaults.

In the remainder of this section, I examine the properties of the calibrated model. Specifically, I analyze the implications for default, present the business cycle properties, and study the dynamics around the default episodes. Since some model properties are similar for the two calibrations, I show them for only the Argentine economy and present the results on the Italian economy in the last subsection.

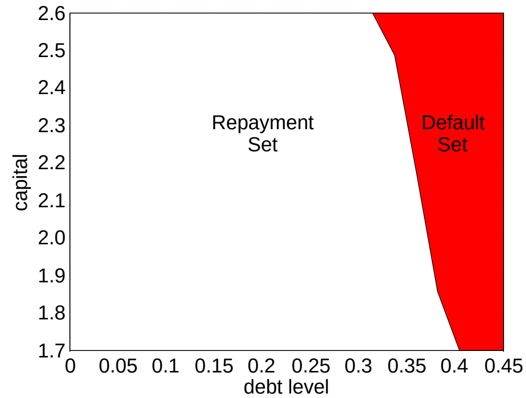
1.4.1 Policy Functions

To highlight the role of different elements in the model, I analyze the government's optimal policies in this subsection. Since the policy functions have the same properties, I show them for the case of Argentina only. I begin with the sovereign's default decisions. Figure 1.3 shows the default and repayment sets of the economy for different combinations of capital stock and debt across different realizations of the productivity shock. Figure 1.3a depicts a low TFP shock, while Figure 1.3b and Figure 1.3c depict the mean and a high shock respectively. Across all figures, the x-axis is the debt level and the y-axis is the capital stock level. Both the debt and the capital stock are normalized as the ratio to output. The shaded region represents all combinations of debt and capital for which the economy is better off defaulting (the default set). The complement region represents all combinations for which repaying the debt is better (the repayment set).

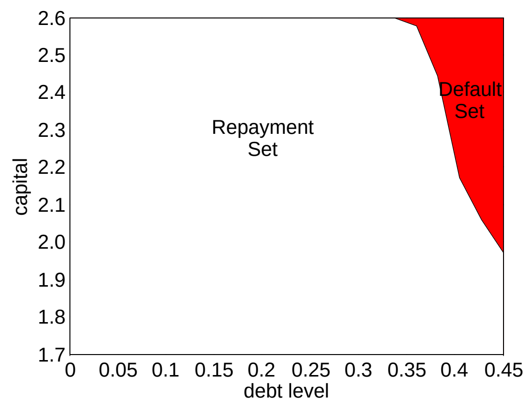
Looking across the subfigures from (a) to (c), we observe the default set shrinks as the TFP z increases. This result is in line with the empirical evidence where countries default more often in bad times. When the TFP is low, the cost of repaying the debt is higher because the government has to levy more taxes to finance government spending and repay the debt. As taxes are distortionary in the model, increasing tax rate during downturn is more harmful to the economy because the distortion cost is convex. Furthermore, the collateral constraint is less binding when z is low because the amount of working capital required is smaller. This translates into a smaller default cost for lower values of z. Altogether, higher cost of repaying and lower cost of defaulting induce the government to default more often when the economy is hit with negative shocks. In general, models with endowment economies have to assume an asymmetric default cost in order to produce the negative correlation between the default risk and output. In my model, however, the endogenous cost, due to the collateral constraint and the distorting taxes, naturally creates an asymmetric default cost.

Figure 1.3 shows a positive correlation between the default risk and capital stock conditional on a given level of TFP and debt. In the model, firms can use both capital and government bonds as collateral. For a given level of bond holdings, a higher level of capital implies there is a larger amount of collateral. This relaxes the collateral constraint and tames the severity of the contraction following default. Thus, ceteris paribus, a higher capital stock makes default more likely. This result may be somewhat surprising, as one may think that economies with more capital stock are wealthier and hence are able to sustain more debt. [19] However, if we compare developed economies and emerging economies (rich and poor countries), they differ much more in their levels of TFP than in the levels of capital stock. More importantly, if we focus only on emerging economies, output and investment are typically growing above trend until a year before a default. In response to high productivity shocks, countries invest more and build up a higher capital stock. They are then more likely to default if there is

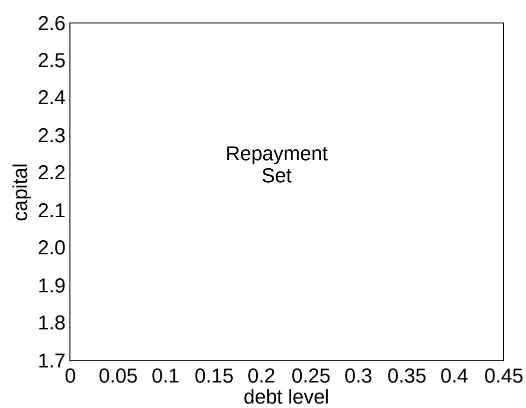
[19] Gordon and Guerron-Quintana (2013) claim that default is more likely to occur in economies with lower capital stock, because these economies have less income to repay their debts. However, in their model, default costs are exogenous and debt does not affect the productive capacity of the economy.



(a) Low productivity shock



(b) Mean productivity shock



(c) High productivity shock

Figure 1.3: Default and Repayment Sets

a large drop in productivity. My model gives a clear explanation for this, because the default cost is smaller if the economy has higher capital.

The model predicts a negative correlation between the default risk and debt level conditional on a given level of TFP and capital, as depicted in Figure 1.3. This is a standard result because higher debt requires the government to charge higher taxes, which gives the government greater incentives to default.

Figure 1.4 depicts the value functions (welfare) of repaying and defaulting. The x-axis is the debt level while the y-axis is the value function. The solid line is the value of repaying and the dotted line is the value of defaulting. The panels, from left to right and from top to bottom, are four combinations of low z and low k, low z and high k, high z and low k, high z and high k respectively. The value functions increase in both the z and k dimensions. The higher the TFP or the higher the capital stock, the higher are the welfare in the economy.

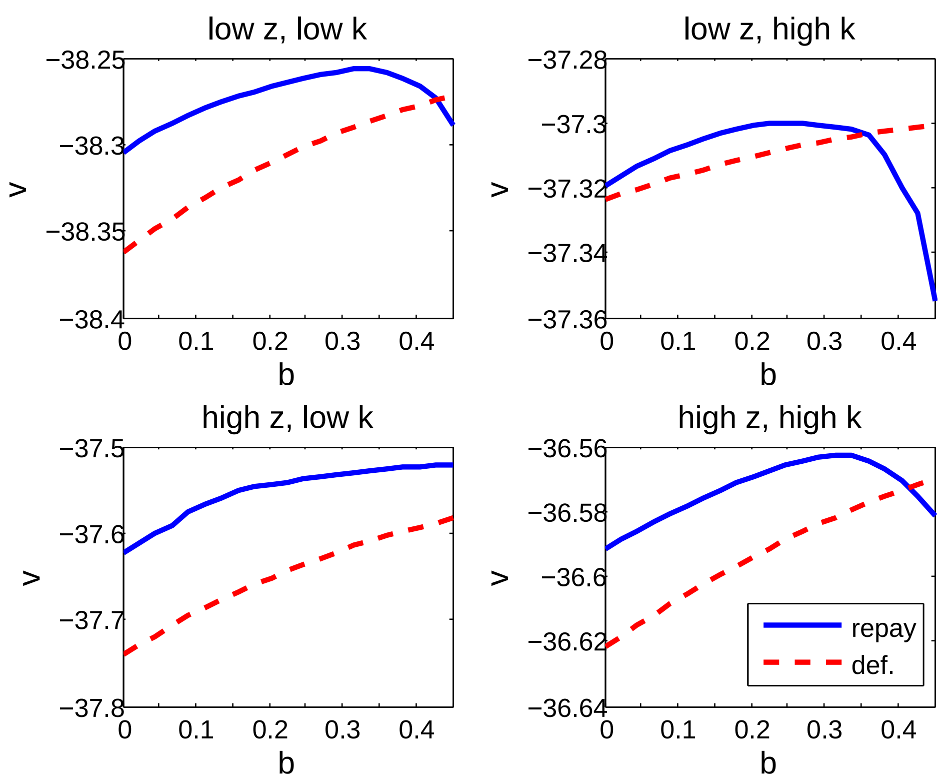


Figure 1.4: Value Functions

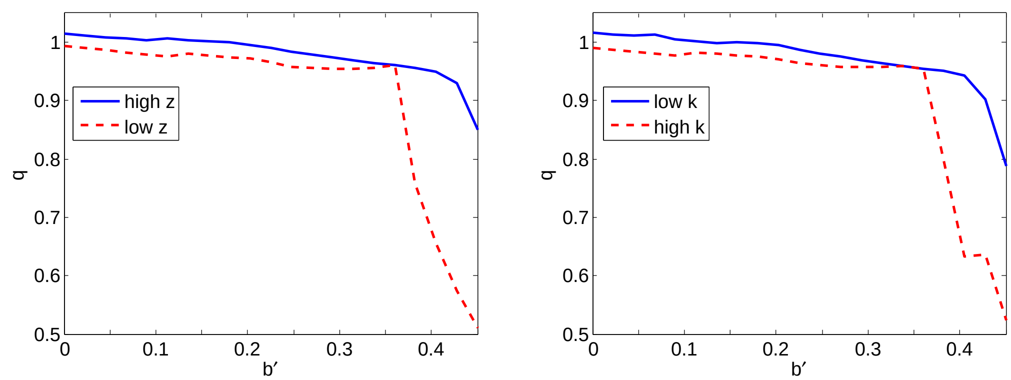
Figure 1.4 also illustrates the correlation between the value function and the debt level. For a given level of z and k, there is a unique level of debt that gives the highest welfare to the economy. This is a breakdown of the Ricardian equivalence, which states that the level of the debt in the economy is irrelevant and does not affect welfare. In the model, increasing debt relaxes the collateral constraint, allowing firms to finance more working capital and increase labor demand. But it implies more distorting taxes, which suppress the labor supply. The benevolent government issues debt to strike a balance between the level of finance friction and the level of tax distortion in the economy. Furthermore, the value of defaulting also depends on the level of the outstanding debt. Due to the assumption of partial default, the outstanding debt is not completely wiped out after default, so the level of debt matters even if the government defaults. In comparison, debt position becomes zero in models with full default, leading to a value of default that is independent of the debt. Empirically, the debt level is important even if the government defaults. For example, Sturzenegger and Zettelmeyer (2008) find that there is a wide variation in haircuts and larger haircuts are associated with larger output drops.

To understand the default incentives better, consider the debt level that yields the highest welfare conditional on not defaulting. Figure 1.4 shows that the welfare-maximizing debt level increases in z and decreases in k. The reason is as follows: when TFP is high, firms want to increase production and are in need of more working capital and more collateral, so government bonds become more valuable. However, when capital is high, firms have relatively more collateral, thus government bonds become less valuable as an alternative form of collateral. Hence, government bonds are less useful to finance working capital when z is low and k is high, which explains the default incentives are the highest in these states.

Figure 1.5 depicts the bond price q as a function of the next period debt b'. The left panel shows it for different values of productivity z while the right panel shows it for different values of capital k. Recall that bond is priced according to

$$q = \frac{\beta}{1-\xi\mu}\mathbb{E}\left[\frac{U_{c'}}{U_c}\left(1-D'\lambda\right)\right].$$

There are two major departures from the bond pricing schedules in the sovereign default literature. First, the model features risk averse pricing while the literature typically assumes risk neutral pricing. As Lizarazo (2013) has shown, risk averse pricing is more relevant empirically and incorporating it in the model explains bond prices better than models with risk neutral investors. Second, there is a liquidity premium to hold bonds, represented by $\frac{1}{1-\xi\mu}$ in the above equation. The liquidity premium arises due to the fact that bonds help to relax the collateral constraint. Bond prices are higher when the collateral constraint is more binding ($\mu > 0$). This implies that firms are willing to purchase bonds at a higher price (lower rate of return) when the collateral level is low, because bonds are useful in relaxing the collateral constraint.



(a) across z

(b) across k

Figure 1.5: Bond Pricing Function

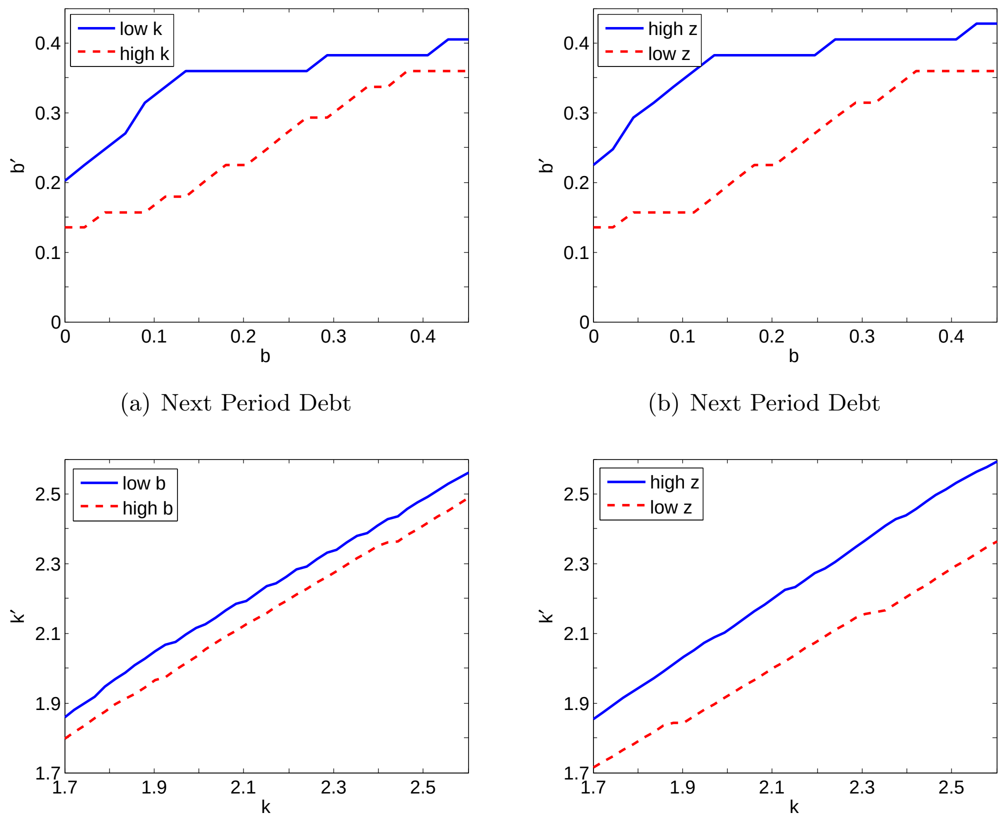
In the left panel of Figure 1.5, ceteris paribus, bond price q is higher for higher z. This is because higher z implies higher consumption c and lower marginal utility of consumption today (lower U_c) relative to tomorrow. High consumption today means households want to save more to smooth consumption. This increases the stochastic discount factor, which in turn increases the price of bond. For high values of b', default risk kicks in and reduces the price of bond. Default risk is much higher for lower z, therefore q decreases at a greater rate. In the right panel, bond price is negatively correlated with capital. This is because low capital k implies firms require more collateral, which increases the value of bond. Mechanically, the collateral constraint is more binding for lower capital stock. This increases the Lagrange multiplier μ (or the liquidity premium of bond) and thus increases bond price q. Again, when b' is high, the default risk channel dominates, and q decreases. It decreases more sharply for higher k due to the positive correlation between capital and default probability.

Figure 1.6 illustrates the evolution of aggregate endogenous state variables. In the upper panels, Figure 1.6a and 1.6b depict the evolution of aggregate debt b. The upper left panel shows the choice of next period debt, conditional on not defaulting, for a high and a low capital and a mean level of z. The next period debt level b' is higher if the current capital k is lower. This is because when there is a small k, the economy is in need of more collateral. Increasing private investment too much will decrease the return to capital. Thus the government increases debt to fill in the gap and alleviate the financial friction in the economy. When k is high, the need for collateral is less pressing, so the tax distortion incentive weighs more and the government does not increase debt as much. The upper right panel shows the choice of next period debt for a high and a low TFP shock and an intermediate level of k. When z is high, firms want to hire more labor and finance more investment. The government has incentive to increase debt and relax the collateral constraint. Furthermore, debt to output ratio is lower for higher z, thus the tax distortion channel is less important and higher debt are sustained.

Figure 1.6c and 1.6d depict the evolution of aggregate capital stock k. The lower left panel shows the choice of next period capital for a high and a low debt level and a mean level of z. Next period capital stock is lower for a higher current level of debt. This shows that debt crowds out capital. The reason is as follows. When the current debt b is higher, the government issues higher level of new debt b', as shown in the upper panels. Higher b' implies that there are less resources devoted to private investment, thus lowering the next period capital stock. Ceteris paribus, this is bad for the welfare. But higher debt translates into higher return to capital, so the overall welfare depends on the state of the economy and the equilibrium conditions. The lower right panel shows the choice of next period capital stock for a high and a low TFP shock and an intermediate level of b. As productivities are persistent, a high productivity today implies a bigger probability of high productivity tomorrow. With higher return to capital, firms increase investment to catch these opportunities. Hence, k' is higher when z is higher.

1.4.2 Cyclical Movements

To assess the business cycle properties of the model, I compare moments from the data with moments from the model's simulations. With shocks drawn from the stationary



(a) Next Period Debt (b) Next Period Debt

(c) Next Period Capital (d) Next Period Capital

Figure 1.6: Evolution of State Variables

distribution of z, I conduct 1000 simulations, each with 500 periods and truncate the first 100 observations.

Table 1.2 shows the statistics related to defaults. According to Reinhart and Rogoff (2011b), domestic defaults are about 1/4 as frequent as external defaults. [20] As the probability of (total) defaults has been estimated to be between 2.5% and 3%, [21] the probability of domestic defaults is about 0.6%. In the model, the default rate is 0.48%, slightly less than the data but reasonably close, as I abstract from other incentives to

[20] Based on Reinhart and Rogoff (2011b), since 1800, there has been 68 domestic defaults and 250 external defaults.

[21] Researchers typically use the fact that Argentina defaulted five times since its independence in 1816 or three times in the past one hundred years, translating into a 2.5% or 3% annual default rate.

default. [22]

Table 1.2: Default Episodes

	Data	Model
Default rate	0.6%	0.48%
Mean output drop	11%	10%
Mean investment drop	36%	32%
Correlation btw default and GDP	-0.11	-0.083
Fraction of defaults with large recessions	32%	26%
Fraction of defaults with GDP below trend	62%	76%

In Table 1.2, the mean output drop during default is 10% in the model, a close match to those observed in the data, which is 11%. On average, investment decreases by about 32% in the model, while it is 36% in the data. The model is able to match these decreases in output and investment very well. These decreases are the endogenous default costs in the model. In both the data and the model, the fall in investment is more than three times that of the drop in output. This again highlights the importance of incorporating investment and capital in the model, which the previous default literature has not done.

Table 1.2 also reports the relationship between output and default based on the historical cross-country data listed by Tomz and Wright (2013) and Benjamin and Wright (2013). The correlation between defaults and output in the model (-0.083) is a good match to the correlation in the data (-0.11). The negative signs indicate that defaults occur in "bad times". This also holds true for "unusually large" recessions, defined as when GDP is two standard deviations below trend. In the data, the fraction of defaults that occur in large recessions is about 32%, while it is 26% in the model. Tomz and Wright (2013) also emphasize that although defaults tend to happen in "bad times", it is not always the case. They report the fraction of defaults that occur with GDP below trend is 62%, implying that in 38% of the times, defaults happen with GDP above

[22] For example, Tomz and Wright (2013) describe political-economic reasons as additional incentives for governments to default. D'Erasmo and Mendoza (2013) claim redistribution as an incentive to default.

trend. The model is able to generate defaults when outputs are above trend, with the fraction of defaults that occur with GDP below trend is 76% in the model, a bit higher than that in the data.

The model economy is able to sustain a debt to GDP ratio up to 35%, with reasonable default rate and discount factor ($\beta = 0.95$ in the model). This ratio is much higher than the ratios typically obtained in the sovereign default literature. For instance, with an endowment economy, Arellano (2008) reports a mean debt to output ratio of only 6%. While Aguiar and Gopinath (2006) obtain a ratio of 27%, they have a very low discount factor of 0.8 and generate an annual default rate of only 0.08%. [23] With a production economy, Mendoza and Yue (2012) report a debt to GDP ratio of 23%, but have a low quarterly discount factor of $\beta = 0.88$. The reason for sustaining a much higher debt level in this model is due to the positive role of the debt in the model economy. In most other models, debt acts as the (only) means to smooth consumption but does not otherwise affect output. In this model, debt provides additional collateral to firms. As explained in previous sections, in addition to capital, debt helps firms to raise more working capital and thus facilitates output production. Furthermore, debt also allows firms to carry wealth across periods without suppressing the returns to capital. Defaulting on debt disrupts these mechanisms and hurts the economy, thus the model gives rise to a much higher debt ratio.

Table 1.3 reports the moments from the Argentine data with those produced by the model. Overall, the model matches output very well. Faced with the same TFP processes, the standard deviation of output (5.19%) in the model is only slightly less than that in the data (5.66%). Different from most of the sovereign default literature, I do not calibrate the model to match the output, but nevertheless, it produces a very close match. In the model, due to risk aversion in the utility function, the government has a strong incentive to smooth consumption. Thus, the most significant reduction in volatility is in consumption relative to output. A stylized fact in emerging economy business cycle is that consumption variability exceeds output variability ($\sigma_c/\sigma_y = 1.14 > 1$). However, in the model, it is reduced to 0.44%, indicating that the government is employing optimal fiscal policies to reduce volatility. This is also a feature of the real

[23] One exception is Chatterjee and Eyigungor (2012), who study long-term debt and produce debt/GDP ratio over 50%.

business cycle model where it generates low consumption volatility relative to output volatility (for example, see Backus, Kehoe, and Kydland (1992)). [24] Due to the decrease in volatility, the correlation between consumption and output (0.52) is much lower in the model, in contrast to 0.89 in the data.

Table 1.3: Simulation Results for Argentina

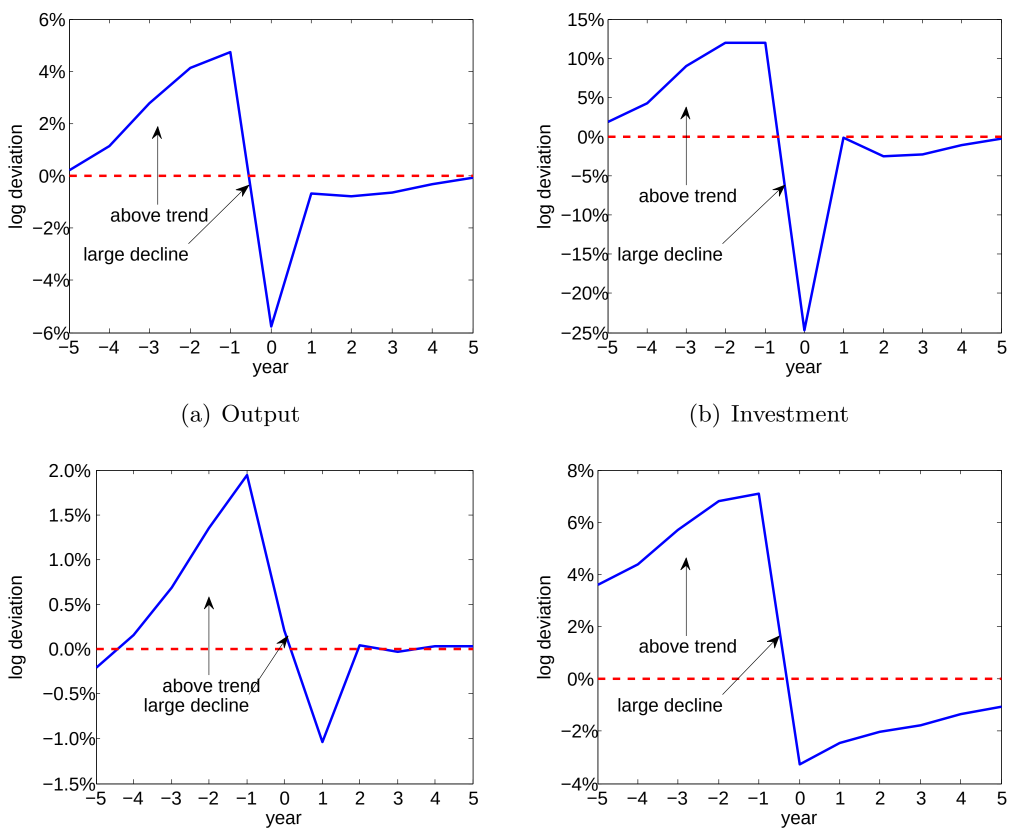
	Data	Model
σ_y	5.66%	5.19%
σ_c/σ_y	1.14	0.44
σ_i/σ_y	2.95	3.38
σ_n/σ_y	0.31	0.51
$corr(y,c)$	0.89	0.52
$corr(y,i)$	0.87	0.95
$corr(y,n)$	0.36	0.63

With capital as a means of production and a form of collateral, the government uses fiscal policies to exploit investment to smooth out consumption. In the data, investment is much more volatile than output (2.95) and there is a tight correlation between investment and output (0.87). In the model, these properties are reinforced, with a volatility ratio of 3.38 and a correlation of 0.95. When the TFP shock is high, the government induces investment to increase by a large amount. This has two implications. First, it does not increase consumption as much and thus reduces consumption volatility. Second, it increases capital stock so that the economy has more buffer to withstand negative shocks in the following periods, therefore increases consumption in the periods when TFPs are low. Besides investment, the government also uses policy to induce more volatility in labor relative to output (0.51) and a closer correlation between output and labor (0.63).

[24] It is a puzzle in international economics that emerging economies have high consumption volatility relative to output volatility and models in general do not have that property. Some literature, for example Arellano (2008), produce that feature because they do not allow for capital and investment, which takes away an important means of consumption smoothing. In the data, investment is usually three to four times more volatile than output.

1.4.3 Dynamics around Default Events

In order to analyze the behavior of the model's dynamics around sovereign default, I compute the average deviations of output, investment, consumption and productivity from their respective trends across default occurrences in the simulated time series data. First, I take log of the simulated data and hp-filter them to get log deviations; second, I identify the simulation periods where default happens; third, I construct a time series of five years before and five years after each default; fourth, I average across default episodes to construct a series of mean log deviations from trends. Figure 1.7 shows the results, with the default periods normalized to date 0.



(a) Output

(b) Investment

(c) Consumption

(d) TFP

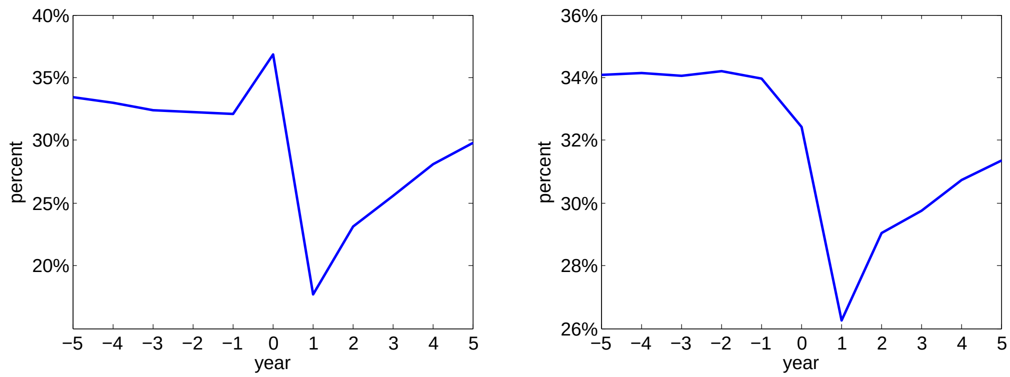
Figure 1.7: Default Dynamics

The model is able to endogenously account for the declines in output, investment and consumption around default episodes, triggered by the declines in TFP. Qualitatively, it is able to deliver the v-shaped behavior of all three variables around defaults, which is consistent with data. The mean declines in output and investment are about 10% and 32% respectively, very close to the empirical counterparts, which are 11% and 36% respectively. Furthermore, in line with data, the model produces investment deviations from trend of magnitude about three times that of the output. When the economy encounters negative shocks, investment decreases in order to smooth out consumption. Despite the significant decline in savings, consumption decreases as well but in a much smaller proportion relative to the drops in output and investment. Decline in consumption, however, is much smaller in the model (2%) than in the data (15%). This requires some explanations. In the Argentine data, while output dropped after default, government spending did not decrease much and net export increased a lot. This implies that consumption has to decrease a lot. However, in the model, I assume a constant government spending to output ratio, which implies a highly procyclical government spending. There is also no import and export in the model. This translates to the result that consumption does not decrease as much.

Furthermore, the model successfully replicates the path leading to default. As illustrated in Figure 1.7, all variables grow above trend until one year before default. This is the general pattern of defaults observed in emerging economies. In the years leading up to the default, countries are typically growing well, building up capital stock through increased investment. In the model, as the economy receives a sequence of positive shocks (refer to Figure 1.7d), firms increase investment to reap the benefits of increased productivity. If the economy then encounters a large negative shock, the cost of default is smaller, because the high level of capital stock built up in the previous periods implies that there is sufficient collateral in the economy to withstand the negative shock.

To illustrate the strategic behavior of the government more clearly, let us take a look at Figure 1.8, which depicts the dynamics of debt and tax around default. The debt to output ratio reaches the peak in the period of default. These are also periods of low productivity and output. If the government were to repay the debt in these periods, it has to charge higher tax rates. Therefore, tax distortions are high in periods of low productivity/output. Due to the concavity of utility function, tax distortions are convex, which means reducing distortion has greater benefit when taxes are high.

This gives the government stronger incentives to default. Coupled with a lower cost of default as explained in the previous paragraph, the government thus finds it optimal to default at the moment of low productivity and high capital stock. In Figure 1.8b, we see that the tax rate is reduced in the period of default. It is reduced further in the period immediately following default, as the economy starts off with a lower debt level and is able to borrow again by issuing new debt.



(a) Debt/GDP (b) Tax

Figure 1.8: Debt and Tax

One anomaly in the model is the path of recovery after default. In the data, it takes roughly three to five years for the economy to grow back to the trend. In the model, however, the recovery is much faster and takes only one to two years. The reason is due to the assumption of no market exclusion after default. After default, the economy starts off with a much lower level of debt. With access to credit market, the government has incentive to increase debt, which has two benefits. First, it reduces current tax rate and thus increases households' labor supply. Second, it increases collateral, thereby allowing firms to take more working capital loan and increases labor demand. The combined forces accelerate the economy's recovery rate.

1.4.4 Analysis for the Italian Economy

In this section, I calibrate the model to the Italian economy. The purpose is to shed light on the default risk in a developed economy. This aspect has been traditionally ignored in the literature, but the ongoing European debt crisis highlights its relevance

Table 1.4: Parameters for Italy

Calibrated Parameters		Value	Target Statistics
Household's discount factor	β	0.95	Standard value
Curvature of labor supply	ν	0.5	Frisch elasticity = 2
Disutility of labor	χ	4.3	Steady state hours = 0.32
Capital share in output	α	0.3	Labor income share = 0.7
Capital depreciation rate	δ	0.1	Investment/GDP = 20%
Gov't spending/GDP	g	0.21	Gov't spending/GDP = 21%
Collateral parameter	ξ	0.385	Mean debt/GDP = 59%
Partial default	λ	0.5	Haircut = 50%
Autocorr. of prod. shock	ρ_z	0.92	Autocorr. of TFP = 0.92
Std. dev. of prod. shock	σ_z	0.019	Std. dev. of TFP = 0.019

and importance. Another purpose is to test the robustness of the model in fitting a developed economy.

I collect annual data on national accounts, worked hours and public debt for Italy from 1980-2010. Data on GDP, households' consumption, investment, government expenditure are from the OECD National Accounts. Data on worked hours and labor income share are from the EU KLEMS database. Data on public debt is from the OECD database on central government debt. It has a breakdown of total public debt into domestic and external debt. I find that Italy has an average domestic debt to GDP ratio of 59%. The calibration strategy is the same, and the calibrated parameter values are in Table 1.4.

Table 1.5: Simulated Defaults for Italy

	Model
Default rate	0.01%
Mean output drop	5.2%

Table 1.5 shows the simulation results for Italy. The simulated default rate in Italy

is only 0.01% per year, almost negligible. [25] This is in fact consistent with the experience of Italy, which has never defaulted in the modern history, although it is exposed to some risk in the current European debt crisis. The simulated default rate for Argentina is 0.48%, almost 50 times of that of Italy. One major difference between Argentina and Italy is the TFP process. The TFP volatility in Argentina is 4.4%, while it is 1.9% in Italy. [26] As the Argentine economy is much more likely to experience very negative shocks, it increases the probability of default, because the government has stronger incentives to default when the economy is in the bad state. In turn, with higher default probability, Argentina sustains a much lower debt ratio of 25% compared to Italy's 59%. In this model, different TFP processes in two countries, coupled with the default cost generated endogenously through the presence of the collateral constraint, are able to jointly account for higher (lower) default rate and lower (higher) debt ratio in Argentina (Italy).

In the current European debt crisis, several of the major concerns for the policy-makers and economists are the spread of default risk and (potential) disruptions to the real economy if involved countries default. I use the model to conduct a counterfactual analysis on what would happen to the Italian economy if it defaults. In Table 1.5, it shows that Italy would suffer an output drop around 5.2% if it defaults. This situation is depicted in Figure 1.9. More than 5% drop in real GDP for a developed country is substantial. This may help to explain why Italy has not yet defaulted and apprehensions in Europe are not without reason.

1.5 Conclusion

This paper develops a model of sovereign default in a production economy with financial frictions, within which output and default risk are jointly determined. The framework allows it to simultaneously examine the behaviors of output and investment, and their interactions with sovereign debt and default. In the model, firms face collateral constraints to finance working capital, which they use to hire workers and make production

[25] A default rate of 0.01% per year translates to once every 10,000 years.

[26] Emerging economies in general encounter much more volatile exogenous shocks than developed economies, regardless the natures of shocks. For example, see Aguiar and Gopinath (2007) and Neumeyer and Perri (2005).

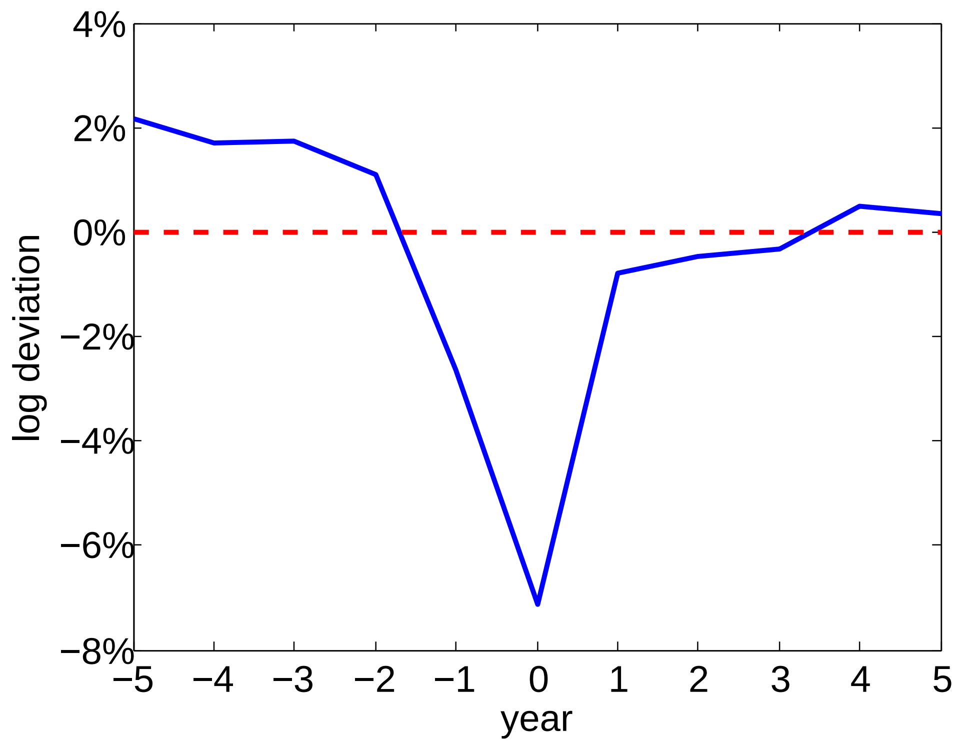


Figure 1.9: Output Drop if Italy Defaults

plans. Both capital and government bonds can be used as collateral. The government chooses tax and debt policies without commitment. If the government defaults, it tightens firms' collateral constraints, forcing firms to reduce labor and cut back on production. But paying back outstanding debt requires the government to levy distorting taxes. The government weighs the costs of reducing output and the benefits of lowering tax distortion when it makes the optimal default decisions.

Quantitatively, the paper is able to account for three stylized facts that have been difficult to capture in the literature. First, a high level of debt (debt to GDP ratio of 25% for Argentina and 59% for Italy) can be sustained in the economy. This is due to the positive role of debt in the model. Government bonds serve as collateral and defaulting on debt makes it more difficult for firms to finance working capital. Furthermore, as defaulting on debt makes the collateral constraint more binding, it also distorts the intertemporal condition, resulting in suboptimal capital accumulations.

Second, the government is more likely to default if the economy experiences a sequence of good productivity shocks followed by a large negative shock. After a sequence of good shocks, the economy ends up with a higher level of capital stock, as firms increase investment in response to high productivity shocks. Cost of default becomes smaller as higher capital means larger amount of collateral. What triggers default then is a drop in productivity, because tax burden becomes higher when productivity/output is low. Third, as the result of the mechanisms described, this paper endogenously generates declines in output and investment around default episodes.

There are a number of interesting and possible extensions to the paper. First, endogenizing haircut decisions can help to explain heterogeneous default events. Empirical work has shown that there is a wide range of haircuts in sovereign defaults and higher haircuts are associated with higher output losses. Studying haircut choices can help to understand what conditions induce the governments to take higher haircuts on debt. Second, allowing for capital income tax would enrich the government's fiscal instruments and connect the paper to the vast literature on optimal labor and capital taxation. Understanding the tradeoffs between default, labor tax and capital tax would allow for a more complete evaluation of the fiscal responses of the government. A third extension is to consider an open economy. Adding this dimension helps to account for both domestic and external public debt and generate implications for international capital flows. While these issues are challenging, I plan to address them in future work.

Chapter 2

Time-Consistent Unemployment Insurance over Business Cycle

2.1 Introduction

[1] Unemployment insurance (UI) program is an important component of social welfare program in many countries. It is designed to help jobless workers smooth consumption while they look for jobs. But the UI program also reduces the incentives of unemployed workers to search for jobs. The generosity of UI benefits is thus a subject of active policy debate. The question has gained more attention since the 2007-2009 recession when unemployment rate reached 10%, [2] and the generosity of the UI program reached an unprecedented level. [3] It has been suggested that the long periods of generous benefits has contributed to sustained high unemployment rate.

This paper argues that because the government cannot commit to a prescribed path for UI benefits during recession, it cannot use UI policy to induce recovery. As a result, the economy suffers from long periods of high unemployment. In the paper, we characterize two economies. The first is governed by a government with full commitment over future UI policies, while the government in the second economy, likely the government in reality, lacks such commitment.

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[1] This chapter is co-authored with Zoe Leiyu Xie.

[2] This is the second highest in postwar U.S. history.

[3] The maximum potential duration an unemployed worker can receive UI benefits was gradually increased from 26 weeks during normal times to 99 weeks.

The model integrates risk-averse workers and endogenous search intensity by the unemployed into the Diamond-Mortensen-Pissarides framework, with shocks to aggregate labor productivity driven by business cycles. Three types of entities inhabit the model economy: workers, firms and the government. Employed workers work for firms and consume their wage income. Unemployed workers receive unemployment benefits and choose how much search effort to exert. Matched firms produce and pay workers wage, while unmatched firms post job vacancies at a cost. Wage is determined through a Nash bargaining process. The government decides on unemployment benefits and taxation. More generous future unemployment benefits reduce unemployed worker's current search intensity. In addition, through general equilibrium effect, higher benefits also reduce firm's vacancy posting by increasing worker's outside option in the wage bargaining process.

We compare two economies. In the first economy, the optimal state-contingent UI policy is the solution to the Ramsey problem of the government, which takes competitive equilibrium conditions as constraints. As the government can commit to future policy, it optimally chooses the level of unemployment benefits for all periods of time and for all possible realizations of productivity shocks. The Ramsey policy, however, is not time consistent. In particular, the current government would like to increase benefits now to help workers smooth consumption, and decrease benefits in the future to encourage workers to search. Without government commitment, this policy cannot be implemented.

In the second economy, we characterize and solve for the time-consistent UI policy in the Markov perfect equilibrium. The government trades off the benefits and costs of unemployment insurance, taking future governments' policies as given. As the government can only choose policy for the current period, the government's decision differs from the Ramsey government in two respects. First, it does not consider how its policy affects the economy in the previous period, while the Ramsey government internalizes this effect. Second, the Markov government can only indirectly influence future polices through the states of the economy, while the Ramsey government predetermines a full sequence of state-contingent policies at time 0.

We calibrate the Markov equilibrium to the U.S. economy. This choice of target is motivated by the view that the Markov economy is a description of the reality. Overall, the Markov economy features more generous unemployment benefits, lower search effort, lower job postings and higher unemployment rate than the Ramsey economy. This is not surprising, given that the Markov government has no commitment and wants to increase benefits. This highlights the importance of commitment.

An important result concerns the dynamic responses of different governments. The Ramsey UI benefits are decreasing in both current labor productivity and unemployment rate, whereas the Markov UI benefits are increasing in both dimensions. The intuition is that the optimal benefits are lower in states of the economy where the marginal social benefit of job creation is higher, because lower benefits can encourage more search by unemployed workers and vacancy postings by firms. Marginal social benefit is higher when productivity is higher, because each worker-firm pair produces higher output; it is also higher when unemployment is higher, because the probability of filling a job is higher. As search and vacancy posting are affected by expectation of next period benefit, the Ramsey government uses current period policy to capture these social benefits in the previous period. The Markov government, in contrast, considers the previous period bygone, and so cannot capture these social gains. Instead, the Markov government increases UI benefits when productivity is high, as there is more resources to dispense with; and increases UI benefits when unemployment is high, because the unemployed now represents a larger share of the population.

Because of different policy responses, the Ramsey and Markov economies have very different dynamics. In response to a one-time negative productivity shock, the Ramsey government initially increases and then slowly reduces UI benefits to the pre-shock level. The initial rise is to help workers smooth consumption, while the subsequent fall creates incentive for search and job posting. The adverse impact of higher benefit on job creation in the initial periods is mitigated by the government's commitment to lower benefits in the future. In response to the same shock, the Markov government lowers benefits immediately, because the costs of financing benefits increase when there are fewer resources. Over time, the Markov government gradually increases benefits to the pre-shock level, as the economy recovers. The richer dynamics of the optimal policy reflects the benefit of commitment – because the Ramsey government has commitment over future policies, it can use temporary changes in the UI policy to smooth consumption over the business cycle. As a result, the Ramsey economy experiences relatively quick recovery in unemployment after shock, while the Markov economy undergoes a much slower recovery.

Our paper contributes to the literature of optimal unemployment insurance. By comparing cyclical responses of the optimal and Markov UI policies, we complement the recent literature on optimal UI policy over the business cycle. These include Jung and Kuester (2015), Landais, Michaillat, and Saez (2010), and Mitman and Rabinovich (2015). Our results highlight the importance of commitment in formulating government policies. The dynamic response of the Markov economy generates a slow recovery in unemployment, which is related to the literature on slow and jobless recovery, for example, Stock and Watson (2012), Shimer (2012), and Heathcote and Perri (2015). A review of the relevant literature is given next.

2.1.1 Related Literature

This paper is closely related to two strands of literature: the literature on unemployment insurance and the time-consistent public policy literature.

The literature looking at unemployment insurance dates back to Mortensen (1977), who argues that unemployment insurance reduces search by the unemployed. The majority of this literature since then takes one of two approaches – either from a positive perspective by studying the effects of actual unemployment insurance policy, or through a normative lens looking for an optimal policy. This paper aims to bridge the two approaches by comparing the optimal policy to the policy without commitment. Here we take the stance that the government, when making unemployment insurance policies, is unable to commit to future policies. As a result, there is welfare loss related to the lack of commitment. In this sense, the no-commitment policy we study is both a positive statement and a no-commitment optimal policy.

One of the classic empirical results in public finance is that social insurance programs such as unemployment insurance reduce labor supply. Earlier works include Moffitt (1985) and Meyer (1990), who show a 10 percent increase in unemployment benefits raises the average unemployment durations by 4-8 percent in the United States. Krueger and Meyer (2002) and Gruber (2007), for example, interpret this finding as evidence that unemployment insurance has significant moral hazard costs. Our framework relies on a similar mechanism. When the expected payoff from unemployment is high relative to future wage, the unemployed search less actively. Chodorow-Reich and Karabarbounis (2015) construct a time series of the opportunity cost of employment, and find that the cost is procyclical and volatile over the business cycle.

More recently, Chetty (2008) explores an alternate explanation for the link between unemployment benefits and duration. He argues that unemployment benefits increase cash on hand for the unemployed, and thus reduces search intensity. This effect is stronger for the unemployed who faces tighter liquidity constraint. Because this "liquidity effect" of unemployment insurance has a socially beneficial effect of correcting credit market failure, the truly optimal benefit level should be higher than if such an effect was ignored. For tractability, this paper abstracts from credit market, and so cannot directly control for the "liquidity effect". However, we model the worker's preference as non-separable between consumption and search intensity. In particular, we allow search to depend negatively on current benefit – when benefits are high, the unemployed have more cash on hand, and so search less actively.

Since the Great Recession of 2007-2009, there has been renewed interest in evaluating the quantitative effect of unemployment insurance on unemployment rate and duration. The reason behind the renewed interest is the extension of maximum potential unemployment benefit duration from 26 to 99 weeks at the peak of the recession and afterward. Nakajima (2012a), for example, quantifies the negative incentive effect of the extension as a 1.4 percentage point increase in unemployment rate during 2007-2012 (or approximately 30% of the overall increase). Fujita (2010), using data from the Current Population Survey, obtains an estimate between 0.8 and 1.8 percentage points. Hagedorn, Karahan, Manovskii, and Mitman (2013) use the natural experiment of neighboring counties in two states having similar labor market conditions but different unemployment benefit duration. They find that the more generous unemployment insurance program increased unemployment rate during the Great Recession. Their emphasis is on the macro-effect of unemployment insurance – lower search reduces firm's incentive to post jobs, and thus further reducing job creation. Chodorow-Reich and Karabarbounis (2015), on the contrary, finds that differences in benefits have a small effect on the opportunity cost of employment. Our paper is related to this recent literature by examining unemployment insurance policy during recessions. Different from this literature, we do not take government policy as given. Instead, we explore how the government determines its policy.

The literature studying optimal unemployment insurance has traditionally adopted a principal-agent framework for example, Hopenhayn and Nicolini (1997), Wang and Williamson (2002), Shimer and Werning (2007) and Shimer and Werning (2008). This framework allows for a characterization of moral hazard frictions in the steady state, but becomes untractable when extended to incorporate aggregate shock. This literature typically shows that the optimal benefit should decline with the unemployment duration of an individual worker. For tractability, our paper abstracts from duration-dependent benefits. More recently, Mitman and Rabinovich (2015) and Landais, Michaillat, and Saez (2010) both study the optimal benefits over the business cycle in a search and matching framework with endogenous unobservable search intensity. Jung and Kuester (2015) take a more general approach by studying the optimal mix of unemployment benefits, hiring subsidies and layoff taxes in a recession. These papers, however, all assume that the government is able to commit to future policies. While this assumption is innocuous and standard from the perspective of normative analysis, such policies are hardly implementable because no real government can credibly plan and stick to a prescribed sequence of policies. Our paper complements these papers by characterizing a policy without commitment, and comparing such policy to the optimal benefits policy over the business cycle. An important message in our paper is that the lack of commitment drastically alters government's policy responses to a recession.

This paper is also related to the literature on time-consistent public policy. [4] Methodologically, our paper follows Klein, Krusell, and Rios-Rull (2008) to characterize the Markov perfect equilibrium of a dynamic game in terms of a generalized Euler equation (GEE). However, while they focus on a deterministic economy, we are interested in how government policy responds to business cycle fluctuations. Recent applications of the concept of Markov perfect equilibrium include Song, Storesletten, and Zilibotti (2012), who study intergenerational conflict over debt in a politico-economic environment.

The rest of the paper proceeds as follows. Section 2.2 describes the model environment and defines the private-sector competitive equilibrium. Section 2.3 presents the Ramsey government and the Markov government. We characterize the solutions and

[4] See Klein and Rios-Rull (2003) for a detailed review of the literature.

solve both governments' problems in this section. Section 2.4 describes the calibration strategy. Section 2.5 presents the quantitative results and discusses alternative scenarios. Section 2.6 concludes. We relegate derivations, sensitivity analysis and additional plots to Appendix B.

2.2 Model

In this section, we describe the model environment and characterize the competitive equilibrium. The model is based on a Diamond-Mortensen-Pissarides model with aggregate productivity shocks.

2.2.1 Environment

Time is discrete and infinite. The model is inhabited by a mass of infinitely lived workers and firms. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Workers are risk-averse and maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t)$$

where $\mathbb{E}_0$ is the period-0 expectation factor, β is the time discount factor. Period utility $U(c, s)$ takes consumption of goods c and search intensity s as inputs. Utility is increasing in c and decreasing in s. Only unemployed workers supply positive search intensity, i.e. there is no on-the-job search. Each period, an employed worker gets paid wage from production. Wages are determined through a canonical bargaining process to be specified later in the section. An unemployed worker receives unemployment benefits b. In addition, an unemployed worker also produces h, which we take as the combined value of leisure, home production and welfare. There is no private insurance markets and workers cannot save or borrow.

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor β. A firm can be either matched to a worker (and producing) or vacant. A vacant firm posting a vacancy incurs a flow cost κ.

Unemployed workers and vacancies form new matches. Let u and v denote the measure of unemployed worker and the measure of vacancies posted respectively. Then the number of new matches formed in a period is given by the matching function $M(su, v)$, where the quantity su is the measure of efficiency units of search by the unemployed in the economy. The matching function exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and is bounded above by the number of potential matches : $M(su, v) \leq \min\{su, v\}$. The job-finding probability per efficiency unit of search intensity, f, and the job-filling probability per vacancy, q, are functions of labor market tightness, $\theta = v/(su)$. More specifically,

$$\begin{aligned} f(\theta) &= \frac{M(su, v)}{su} = M(1, \theta) \\ q(\theta) &= \frac{M(su, v)}{v} = M\left(\frac{1}{\theta}, 1\right) \end{aligned}$$

Following the assumptions made on M, $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ. The job finding probability for an unemployed searching with intensity s is $sf(\theta)$. Existing matches are destroyed exogenously with constant job separation probability δ.

Only a matched pair of a worker and a firm can produce. Each matched pair produces z, where z is the aggregate labor productivity. z is constant $\bar{z}$ in the steady state, and time-varying z_t in the economy off steady-state.

2.2.2 The Government

The government cannot borrow or lend; instead it balances budget each period. The government finances unemployment benefits b through a lump sum tax, τ, on all workers, both employed and unemployed. The government budget constraint is

$$\tau = ub \tag{2.1}$$

The government decides on the generosity of the unemployment insurance program by varying benefit level, $b \geq 0$. Once a benefit level is determined, all unemployed workers receive the same benefit in that period. This way of modeling the unemployment insurance system is a simplification of the reality where not all unemployed workers receive benefits. This assumption is common in the literature, for example, Landais, Michaillat, and Saez (2010) and Jung and Kuester (2015) both assume all unemployed receive benefits. The advantage of this setup is reduced computational complexity while still allowing the generosity of the unemployment insurance program to change. The unemployment benefits here can be thought of as compounding the potential duration and level of unemployment benefits. [5]

2.2.3 Timing

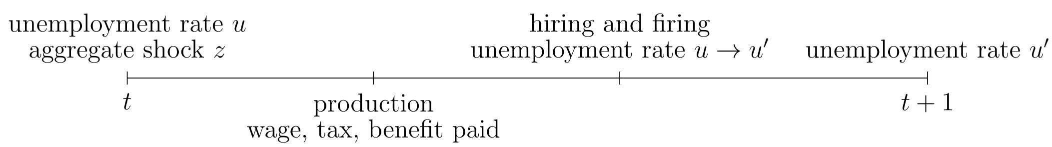

Figure 2.1: Timing of Events

The timing of events within a period is illustrated in Figure 2.1 and is as follows. The economy enters a period t with a level of unemployment u. The aggregate shock z is then realized. (z, u) are the aggregate states of the economy. Government policies (b, τ) for the period is known to workers and firms.

Employed workers produce z and receive a bargained wage w. Unemployed workers produce h and receive benefits b. All workers pay a lump sum tax τ out of wage or benefit.

Given aggregate states and government policies for the period, unemployed workers choose search intensity s. At the same time, firms decide how many vacancies to post, at cost κ per vacancy. The aggregate search is then su, and the market tightness is equal to $\theta = v/(su)$. The fraction of unemployed workers who find jobs is $f(\theta)s$. At the same time, a fraction δ of the existing $1 - u$ matches are exogenously destroyed. The law of motion of unemployed workers is

$$u' = \delta(1-u) + (1 - f(\theta)s)u \tag{2.2}$$

2.2.4 Workers

Denote by g the government policy (b, τ). A worker entering a period unemployed consumes $h + b$ and chooses search intensity s. With probability $f(\theta(z, u; g))s$, he finds a job and starts working the following period. Let $V^e(z, u; g)$ and $V^u(z, u; g)$ be the

[5] Equivalently, it can be thought of as compounding benefit level and proportion of unemployed workers on benefit at any time.

values of an employed and an unemployed worker, respectively, with the beginning-of-period unemployment u and realized aggregate shock z, given government policy for that period $g = (b, \tau)$. An unemployed worker's optimization problem is

$$\begin{aligned} V^u(z,u;g) &= \max_s \; U(c,s) + \beta f(\theta(z,u;g)) s \mathbb{E} V^e(z',u';g') \\ &\quad + \beta(1 - f(\theta(z,u;g))s)\mathbb{E} V^u(z',u';g') \end{aligned} \tag{2.3}$$

A worker entering a period employed produces and consumes his wage w. With probability δ, he loses his job and becomes unemployed the following period. There is no intra-temporal search, so a newly separated worker remains unemployed for at least one period. The Bellman equation of an employed worker is then

$$V^e(z,u;g) = U(w(z,u;g),0) + \beta(1-\delta)\mathbb{E}V^e(z',u';g') + \beta\delta\mathbb{E}V^u(z',u';g') \tag{2.4}$$

Notice that market tightness θ and wage w are functions of the economy's states, (z,u). This is because they are objects determined in an equilibrium. As mentioned before, job separation rate δ is taken to be constant through time.

2.2.5 Firms

In order to be matched with a worker and produce, a firm posts a vacancy. [6] A firm that posts a vacancy incurs a flow cost κ. With probability $q(\theta(z,u;g))$, a vacancy is filled and ready for production the following period. Let $J^u(z,u;g)$ be the value of an unmatched firm posting a vacancy. The Bellman equation of an unmatched firm is

$$\begin{aligned} J^u(z,u;g) &= -\kappa + \beta q(\theta(z,u;g))\mathbb{E}J^e(z',u';g') \\ &\quad + \beta(1 - q(\theta(z,u;g)))\mathbb{E}J^u(z',u';g') \end{aligned} \tag{2.5}$$

where $J^e(z,u;g)$ is the value of a matched firm. In equilibrium, under free-entry condition, the firm will post vacancies $v(z,u;g)$ until $J^u(z,u;g) = 0$.

A matched firm receives output net of wages $z - w(z,u;g)$. With constant probability δ, a match is destroyed at the end of period. The Bellman equation of a matched firm is

$$J^e(z,u;g) = z - w(z,u;g) + \beta(1-\delta)\mathbb{E}J^e(z',u';g') + \beta\delta\mathbb{E}J^u(z',u';g') \tag{2.6}$$

[6] The firms can be viewed as a representative firm with a collection of jobs and posts several vacancies.

2.2.6 Wage Determination

Vacant jobs and unemployed workers are randomly matched each period according to the aggregate matching function $M(su, v)$. A realized match produces some economic rent that is shared between the firm and the worker through Nash bargaining. The assumption of Nash bargaining allows comparison with the literature. We assume that wages are set period by period, so equilibrium wages respond to the state of the economy.

Worker's surplus is the difference between the values of working at wage w and being unemployed and receiving benefit b. As a result, higher benefits increase worker's surplus, and tends to drive up bargained wage. Firm's surplus is the difference between the value of a match and that of running a vacancy. As explained before, vacant firm posts vacancies until its value is zero. Thus, the firm's outside option is zero in equilibrium.

In particular, wage is chosen to maximize a weighted product of the worker's surplus and the firm's surplus when the state of the economy is (z, u) and government policy is $g = (b, \tau)$. The worker-firm pair thus solves

$$\max_{w} \quad \Big(V^e(z, u; g) - V^u(z, u; g)\Big)^{\zeta} \Big(J^e(z, u; g) - J^u(z, u; g)\Big)^{1-\zeta} \tag{2.7}$$

where $\zeta \in (0, 1)$ is the bargaining power of the worker. $V^e(z, u; g) - V^u(z, u; g)$ is the worker's surplus, and $J^e(z, u; g) - J^u(z, u; g)$ is the firm's surplus from the match. The solution to this bargaining problem, denoted $w(z, u; g)$, is a function of the economy's state.

2.2.7 Competitive Equilibrium

Definition 3. (Competitive Equilibrium) Given a policy $g = (b, \tau)$ and an initial condition (z, u), a competitive equilibrium consists of (z, u)- measurable functions for wages $w(z, u; g)$, worker's search intensity $s(z, u; g)$, market tightness $\theta(z, u; g)$, unemployment rate $u'(z, u; g)$, and value functions $V^e(z, u; g)$, $V^u(z, u; g)$, $J^e(z, u; g)$, $J^u(z, u; g)$ such that for all $(z, u; g)$

- the value functions satisfy the worker and firm Bellman equations (2.3)-(2.6)
- the search intensity s solves the unemployed worker's maximization problem of (2.3)

- the market tightness θ is consistent with the free-entry condition, $V^u(z, u; g) = 0$
- the wage w solves the maximization problem of (2.7)
- unemployment satisfies the law of motion equation (2.2)
- tax τ and benefit b satisfy the government's budget constraint (2.1)

2.2.8 Characterization

The competitive equilibrium can be characterized by three optimality conditions. [7] Appendix B.1 contains derivation of the optimality conditions. In what follows, primes denote variables of the following period, and subscripts denote derivatives.

The optimal choice of search intensity s for the unemployed worker is characterized by

$$\begin{aligned} & \frac{-U_s(h+b-\tau, s)}{f(\theta)} \\ = \;& \beta\mathbb{E}\left[U(w'-\tau', 0) - U(h+b'-\tau', s') + (1-f(\theta')s'-\delta)\frac{-U_s(h+b'-\tau', s')}{f(\theta')}\right] \end{aligned} \tag{2.8}$$

The worker's optimality condition states that the marginal cost (left-hand side) of increasing the job finding probability equals the marginal benefit (right-hand side). The marginal cost is the marginal disutility of search of the unemployed worker weighted by the aggregate job finding rate per efficiency unit of search. The marginal benefit is the sum of utility gain from being employed next period and the benefit of economizing on future search cost. A higher future benefit b' reduces the utility gain from being employed the next period, and thus lowers the marginal benefit of search today.

The firm's optimality condition is

$$\frac{\kappa}{q(\theta)} = \beta\mathbb{E}\left[z' - w' + (1-\delta)\frac{\kappa}{q(\theta')}\right] \tag{2.9}$$

where the marginal cost (left-hand side) equals the marginal benefit (right-hand side) of a filled vacancy. The marginal cost is the flow cost of posting a vacancy weighted by the probability of filling that vacancy. The marginal benefit is the profits from employing

[7] To economize on notation, we suppress the dependence on $(z, u; g)$. It should be understood throughout that the optimal decisions are functions with arguments $(z, u; g)$.

a worker. Because a newly formed match does not become operational until the next period, the benefit from production only has components from the next period.

Finally, Nash bargaining implies a relationship between the worker's surplus from being employed and the firm's surplus from hiring a worker.

$$\frac{\left[U(w-\tau,0)-U(h+b-\tau,s)+(1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)}\right]/U_c(w-\tau,0)}{z-w+(1-\delta)\frac{\kappa}{q(\theta)}}=\frac{\zeta}{1-\zeta}$$

The left-hand side of the equation is the ratio of worker's to firm's surplus weighted by marginal utility of higher wage. The worker's surplus (top part) comes from utility gain of being employed and reduced search cost (employed worker searches zero). Because workers are risk-averse, changes in wages have non-linear effect on his utility, as represented by $U_c(w-\tau,0)$. The firm's surplus (bottom part) derives from profit and reduced vacancy posting cost (producing firm posts zero vacancy). The right-hand side of the equation is the ratio of the worker's to firm's bargaining power. Equilibrium wage then equates the weighted ratio of worker/firm surplus to the ratio of their respective bargaining power. Rearranging terms into a more compact condition for the equilibrium wage gives the following equation,

$$\begin{aligned}&\zeta U_c(w-\tau,0)\left[z-w+(1-\delta)\frac{\kappa}{q(\theta)}\right]\\ =\;&(1-\zeta)\left[U(w-\tau,0)-U(h+b-\tau,s)+(1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)}\right]\end{aligned}\tag{2.10}$$

A higher future benefit b' lowers worker's surplus. Given the future Nash bargaining condition, next period's workers demand higher wage w', and thus firm's surplus is lower in equilibrium. The free-entry condition of (2.9) then implies a lower θ, and thus a lower job-finding rate per efficiency search unit $f(\theta)$.

A contemporaneous decrease in the aggregate labor productivity z reduces firm's surplus in (2.10) and as a result wages fall. Because current wage and productivity do not enter worker's and firm's optimality conditions (2.8)-(2.9), the contemporaneous fall in z does not directly affect search or job-finding rate. Now consider a fall in the expected future productivity z'. In this latter case, firm reduces vacancy posting since expected return to future production is lower. Worker reduces search intensity, both because fewer vacancies lead to lower per-search-unit job-finding probability, and because lower expected aggregate productivity implies lower expected future wages.

Because different z' lead to different equilibrium search and job-finding rate, the government will optimally tailor its unemployment benefit policy to current and future economic conditions. In the next section, we analyze how a benevolent government, under assumptions of commitment and non-commitment, designs benefit policy.

2.3 Government Policies

In this section, we describe how the government chooses its policies. We assume the government is a utilitarian planner, who maximizes the expected value of the worker's utility. The government policy instruments include unemployment benefit b and tax τ. We first describe a Ramsey government who can commit to future policies and solve an optimal policy problem. As is well known, the Ramsey solution is in general not time-consistent, which is also the case here. We then describe a Markov government who does not have the ability to commit and solve a time-consistent policy. Lastly, we compare the Ramsey and Markov outcomes.

2.3.1 Ramsey Government

Since the Ramsey government has commitment to all its future policies at the beginning of time, the government's decision problem is therefore to choose a sequence of unemployment benefits and taxes $\{b_t, \tau_t\}_{t=0}^{\infty}$ in order to maximize the worker's utility, taking into account how the private sector will respond to these policies. At time 0, the government decides on its policies for all future periods and for all possible realizations of shocks. The private sector takes government policies as given and follows the timing described in Section 2.2.3.

Equivalently, the government's problem can be written as one of choosing policies $\{b_t, \tau_t\}_{t=0}^{\infty}$, and allocation and prices $\{w_t, s_t, \theta_t, u_{t+1}\}_{t=0}^{\infty}$ to maximize utility subject to the government budget constraint and competitive equilibrium conditions. [8] Formally, the government objective function at time t is given by $R(u_t, b_t, \tau_t, w_t, s_t) = (1-u_t)U(w_t - \tau_t, 0) + u_t U(h + b_t - \tau_t, s_t)$.

Definition 4. (Ramsey Policy) Given an initial unemployment rate u_0 and aggregate labor productivity z_0, the optimal government policy with commitment consists of a

[8] This is the primal approach to Ramsey problem.

sequence of benefits and taxes $\{b_t, \tau_t\}_{t=0}^{\infty}$ that solves

$$\max_{\{b_t,\tau_t,w_t,s_t,\theta_t,u_{t+1}\}_{t=0}^{\infty}} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t R(u_t, b_t, \tau_t, w_t, s_t)$$

over the set of all policies that satisfy the government's budget constraint (2.1), the worker's law of motion equation (2.2) and the competitive equilibrium conditions (2.8)-(2.10), for all time t and aggregate shock $\{z_t\}_{t=0}^{\infty}$.

For easy exposition, we rewrite the competitive equilibrium conditions sequentially and use auxiliary functions $\tilde{\eta_0}$, $\tilde{\eta_1}$, $\tilde{\eta_2}$ and $\tilde{\eta_3}$ to denote the flow equation and the three private-sector optimality conditions (2.8)-(2.10) respectively, [9]

$$\begin{aligned}
\tau_t &= u_t b_t && (2.11)\\
\tilde{\eta_0}(u_t, s_t, \theta_t, u_{t+1}) &= 0 && (2.12)\\
\tilde{\eta_1}(u_t, b_t, \tau_t, s_t, \theta_t, u_{t+1}, b_{t+1}, \tau_{t+1}, w_{t+1}, s_{t+1}, \theta_{t+1}) &= 0 && (2.13)\\
\tilde{\eta_2}(\theta_t, z_{t+1}, w_{t+1}, \theta_{t+1}) &= 0 && (2.14)\\
\tilde{\eta_3}(z_t, u_t, b_t, \tau_t, w_t, s_t, \theta_t) &= 0 && (2.15)
\end{aligned}$$

where the three private-sector optimality conditions play the role of incentive constraints in the optimal policy problem, similar to the incentive constraints in a principal-agent setup, e.g. Hopenhayn and Nicolini (1997).

To derive a set of conditions that characterize the Ramsey policy, we substitute in (2.11) and let $\beta^t\pi^t\lambda_t$, $\beta^t\pi^t\mu_t$, $\beta^t\pi^t\gamma_t$ and $\beta^t\pi^t\nu_t$ be the Lagrange multipliers on (2.12)-(2.15), where π^t is the probability of a history of realizations $\{z_0, z_1, \ldots, z_t\}$ given an initial condition z_0. The optimal government policy can be characterized by the following government's first-order conditions with respect to b_t, w_t, s_t, θ_t and u_{t+1} for all time $t > 0$

$$\begin{aligned}
\mu_{t-1}\frac{\tilde{\eta}_{1b',t-1}}{\beta} + \mu_t\tilde{\eta}_{1b,t} + \nu_t\tilde{\eta}_{3b,t} &= R_{b,t}\\
\mu_{t-1}\frac{\tilde{\eta}_{1w',t-1}}{\beta} + \gamma_{t-1}\frac{\tilde{\eta}_{2w',t-1}}{\beta} + \nu_t\tilde{\eta}_{3w,t} &= R_{w,t}\\
\mu_{t-1}\frac{\tilde{\eta}_{1s',t-1}}{\beta} + \lambda_t\tilde{\eta}_{0s,t} + \mu_t\tilde{\eta}_{1s,t} + \nu_t\tilde{\eta}_{3s,t} &= R_{s,t}\\
\mu_{t-1}\frac{\tilde{\eta}_{1\theta',t-1}}{\beta} + \gamma_{t-1}\frac{\tilde{\eta}_{2\theta',t-1}}{\beta} + \lambda_t\tilde{\eta}_{0\theta,t} + \mu_t\tilde{\eta}_{1\theta,t} + \gamma_t\tilde{\eta}_{2\theta,t} + \nu_t\tilde{\eta}_{3\theta,t} &= 0
\end{aligned}$$

[9] See Appendix B.1 for more details.

$$\lambda_t \tilde{\eta}_{0u',t} + \mu_t \mathbb{E}_t \tilde{\eta}_{1u',t} = \beta \mathbb{E}_t \left\{ R_{u,t+1} - \lambda_{t+1} \tilde{\eta}_{0u,t+1} - \mu_{t+1} \tilde{\eta}_{1u,t+1} - \nu_{t+1} \tilde{\eta}_{3u,t+1} \right\} \quad (RAM)$$

where primes denote next period, and subscripts are derivatives.

The period-t solution is state dependent. It depends on the current productivity z_t and the beginning-of-period unemployment level u_t, as well as multipliers $(\mu_{t-1}, \gamma_{t-1})$. μ is the marginal value of relaxing the optimal search condition for the unemployed worker (2.8), and γ is the marginal value of relaxing the firm's equilibrium free-entry condition (2.9). The presence of μ_{t-1} and γ_{t-1} as states in the optimal policy captures commitment – the Ramsey government in period t has to deliver these marginal values, which it promised for the worker and firm in period $t-1$.

Note that commitment is assumed in the Ramsey case. If given the choice to break promise, the government will deviate from the sequence of policies prescribed by the government at time 0. The government of period t has an incentive to promise low future unemployment benefits to encourage search and vacancy posting, because, as explained in Section 2.2, current search and job-finding probability are higher when the future benefits are expected to be lower. But after employment outcome of period t has been realized, the government has an incentive to smooth workers' consumption by providing high benefits. This incentive to deviate from original plan is what constitutes time inconsistency in the Ramsey problem.

A Simple Example to Illustrate Time Inconsistency

We consider a simple example to illustrate the presence of time inconsistency in the Ramsey problem. There are two periods and a unit measure of workers. Workers search in the first period and consume in the second period. Assume no time discounting and no firms. In the first period, $1-\bar{u}$ of workers are guaranteed a job in the second period. The remaining $\bar{u} = 0.05$ workers choose how much to search, $s \in (0,1)$, for a job starting in the second period. Search incurs utility cost governed by the convex function $v(s)$. Worker's utility of consumption is given by $U(c)$.

With probability s, the worker finds a job and receives wage $\bar{w} = 1$ in the second period; otherwise he receives unemployment benefit $b \in (0,1)$. Optimal choice of search is thus characterized by $v_s(s) = U(\bar{w}) - U(b)$. The number of unemployed workers in the second period is $u = (1-s)\bar{u}$.

Government in this economy chooses b at the beginning of period 1 to maximze average utility

$$\begin{aligned} \max W &= (1-u)U(\bar{w}) + u\left[U(b) - v(s)\right] \\ \text{subject to} \quad u &= (1-s)\bar{u} \\ v_s(s) &= U(\bar{w}) - U(b) \end{aligned}$$

Essentially, the government is solving

$$\max_{s\in(0,1)} \quad [1-(1-s)\bar{u}]U(\bar{w}) + (1-s)\bar{u}\left[U(\bar{w}) - v_s(s) - v(s)\right]$$

with first-order condition given by

$$\bar{u}\left[v_s(s) + v(s)\right] - (1-s)\bar{u}\left[v_{ss}(s) + v_s(s)\right] = 0$$

Let $U(c) = \log(c)$ and $v(s) = \frac{s^2}{2}$. The government optimally chooses $s^* = 0.549$, $b^* = 0.578$ and $u^* = 0.0226$, with average utility $W^* = -0.0158$.

Now suppose the government can revise benefit after workers have chosen s. Then the ex-post optimal policy is $\hat{b} = 1$, with ex-post average utility given by $\hat{W} = (1 - u^*)\log\bar{w} + u^*\left[\log\hat{b} - (s^*)^2/2\right] = -0.0034 > W^*$. In fact, any $\hat{b} > b^*$ will result in higher ex-post average utility. The fact that there exists a better policy ex-post illustrates the time inconsistency in this setup; time inconsistency, in turn, means lack of commitment leads to different policy outcomes than an economy with government commitment.

2.3.2 Markov Government

In this section, we consider government policies that are time consistent. We use the concept of Markov perfect equilibrium, similar to that in Klein, Krusell, and Rios-Rull (2008). By construction, the government policy in such an equilibrium is time consistent.

Intuitively, one can think of the economy as having a sequence of governments, each lasting only one period. Each successive government only chooses current policy, taking future governments' policies as given. It neither considers how its policy affects previous periods, nor can it directly choose policies for future periods. Like Klein, Krusell, and Rios-Rull (2008), we focus on equilibria where government policy depends differentiably on the state of the economy.

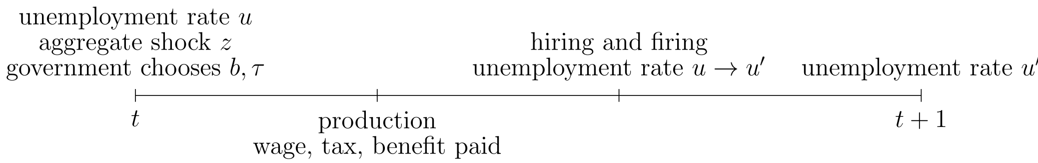

Figure 2.2: Timing of Events under Markov Government

The timing of events is illustrated in Figure 2.2. At the beginning of each period, the government chooses its benefit and tax policy for the current period. The private-sector agents (firms and workers) then move to choose its current period search, vacancy posting and wage, as described in Section 2.2.3. Because the economy consists of a mass of workers and firms, each of measure zero, the private-sector agents take current and future government policies as given.

The equilibrium described above can be equivalently stated as an equilibrium where the government chooses policy and private-sector allocations together given the state of the economy. The government objective function in this case is given by $R(u, b, \tau, w, s) = (1-u)U(w-\tau, 0) + uU(b-\tau, s)$.

Definition 5. (Markov Perfect Equilibrium) A Markov perfect equilibrium consists of a value function G, government policy functions $\{\Psi, \Gamma\}$, and private decision rules $\{W, S, \Theta, \Pi\}$ such that for all beginning-of-period unemployment u and aggregate productivity z, $b = \Psi(z, u)$, $\tau = \Gamma(z, u)$, $w = W(z, u)$, $s = S(z, u)$, $\theta = \Theta(z, u)$ and $u' = \Pi(z, u)$ solve

$$\max_{b,\tau,w,s,\theta,u'} \quad R(u, b, \tau, w, s) + \beta \mathbb{E} G(z', u')$$

subject to

- the government budget constraint (2.1)
- the worker's law of motion

$$\eta_0(u, s, \theta, u') = u' - \delta(1-u) - [1 - f(\theta)s]\, u \tag{2.16}$$

- the private-sector optimality conditions below

$$\begin{aligned}
& \eta_1(u, b, \tau, s, \theta, z', u'; \Psi, \Gamma, W, S, \Theta) \\
:= \quad & \frac{-U_s(b-\tau, s)}{f(\theta)} \\
& -\beta E\left[U(W(z', u') - \Gamma(z', u'), 0) - U(\Psi(z', u') - \Gamma(z', u'), S(z', u'))\right] \\
& -\beta E\left[(1 - f(\Theta(z', u'))S(z', u') - \delta)\frac{-U_s(\Psi(z', u') - \Gamma(z', u'), S(z', u'))}{f(\Theta(z', u'))}\right] \\
= \quad & 0 \qquad (2.17)
\end{aligned}$$

$$\begin{aligned}
\eta_2(\theta, z', u'; W, \Theta) \quad := \quad & \frac{\kappa}{q(\theta)} - \beta E\left[z' - W(z', u') + (1-\delta)\frac{\kappa}{q(\Theta(z', u'))}\right] \\
= \quad & 0 \qquad (2.18)
\end{aligned}$$

$$\begin{aligned}
& \eta_3(z, u, b, \tau, w, s, \theta) \\
:= \quad & \zeta U_c(w-\tau, 0)\left[z - w + (1-\delta)\frac{\kappa}{q(\theta)}\right] \\
& -(1-\zeta)\left[U(w-\tau, 0) - U(b-\tau, s) + (1 - f(\theta)s - \delta)\frac{-U_s(b-\tau, s)}{f(\theta)}\right] \\
= \quad & 0 \qquad (2.19)
\end{aligned}$$

and

- the government value function satisfies the functional equation

$$G(z, u) \quad \equiv \quad R(u, \Psi(z, u), \Gamma(z, u), W(z, u), S(z, u)) + \beta \mathbb{E} G(z', \Pi(z, u))$$

For ease of exposition, we have used the auxiliary functions η_0, η_1, η_2, η_3. Because policies are functions of aggregate productivity and unemployment, the auxiliary functions η_1 and η_2 are functions of next period states z' and u'. In comparison, the Ramsey auxiliary functions $\tilde{\eta}_1$ and $\tilde{\eta}_2$ as defined in (2.13) and (2.14) are simply functions of next period decisions such as b', s'.

Let λ, μ, γ, ν be the Lagrange multipliers on (2.16)-(2.19), respectively. The benefit policy in a Markov equilibrium can be characterized by the following government's first-order conditions with respect to b, w, s, θ and u', [10]

$$\begin{aligned}
\mu\eta_{1b} + \nu\eta_{3b} &= R_b \\
\nu\eta_{3w} &= R_w \\
\lambda\eta_{0s} + \mu\eta_{1s} + \nu\eta_{3s} &= R_s \\
\lambda\eta_{0\theta} + \mu\eta_{1\theta} + \gamma\eta_{2\theta} + \nu\eta_{3\theta} &= 0 \\
\lambda\eta_{0u'} + \mu\mathbb{E}\eta_{1u'} + \gamma\mathbb{E}\eta_{2u'} &= \beta\mathbb{E}\Omega'_u = \beta\mathbb{E}\left\{R'_u - \lambda'\eta'_{0u} - \mu'\eta'_{1u} - \nu'\eta'_{3u}\right\} \qquad (MAR)
\end{aligned}$$

where primes denote next period, and subscripts are derivatives. Note that because η_1 and η_2 contain functions of next period unemployment u' in the form of next period policy functions, derivatives of $\eta_{1u'}$ and $\eta_{2u'}$ contain policy function derivatives.

The Markov perfect equilibrium is then characterized by a system of functional equations (2.1), (2.2), (2.17)-(2.19) and (MAR). An analytical characterization of the Markov perfect equilibrium is not available. We solve for the equilibrium numerically using a standard cubic spline projection method to approximate the policy functions.

The Generalized Euler Equation

To build some intuition for how b is determined, we combine the government first-order conditions into a single equation that characterizes the Markov benefit policy, [11]

$$\begin{aligned}
0 \;=\; & \underbrace{\left[R_b + \mathbb{E}\tilde{W}_b R_w + \mathbb{E}\tilde{S}_b R_s\right]}_{\text{effect of } db \text{ holding } u'} + \beta\mathbb{E}\tilde{\Pi}_b \underbrace{\left[R'_u + \tilde{W}'_u R'_w + \tilde{S}'_u R'_s\right]}_{\text{effect of } du' \text{ holding } u''} \\
& + \beta\mathbb{E}\; \underbrace{\tilde{\Pi}_b\left(-\frac{\tilde{\Pi}'_u}{\tilde{\Pi}'_b}\right)}_{db'/db \text{ holding } u'} \underbrace{\left[R'_b + \tilde{W}'_b R'_w + \tilde{S}'_b R'_s\right]}_{\text{effect of } db' \text{ holding } u''} \qquad (GEE)
\end{aligned}$$

where the functions with tilde are transformations of the ones without, e.g. $S(z,u) \equiv \tilde{S}(z, b(u), u)$ and $S_u = \tilde{S}_u + \tilde{S}_b\Psi_u$. Because of the presence of policy function derivatives

[10] We substitute government budget constraint (2.1) into the rest of the conditions to reduce the number of unknowns.

[11] Appendix B.1 contains two derivations of the GEE, one with policy functions defined as before, e.g. $W(z,u)$, and the other with policy functions such as $\tilde{W}(z,b,u)$. It can shown that the equilibria based on the two definitions are in fact equivalent.

such as $\tilde{S}_u$ and $\tilde{S}_b$, the above equation is also known as the Generalized Euler Equation or GEE. From the GEE, it is obvious any change in b has three effects. First, it affects the contemporaneous wages and search, and thus both directly and indirectly changes the value of current government objective function. Second, through changing u', it changes next period's unemployment, wages and search, thus changing next period's value. Last, it also has an effect on next period's value through its effect on next period benefit b'. The government determines current benefit by setting the net marginal value of b to zero.

Notice that the GEE does not contain explicitly the derivative of Ψ; it appears indirectly in private-sector policy derivatives with respect to b, such as $\tilde{S}_b$. This reflects an important point made earlier – the successive governments agree on a policy rule Ψ. The Markov government does not try to manipulate its successor through changing current b, hence the absence of Ψ_b directly from the GEE. The fact that Ψ_b affects private-sector policy derivatives captures the fact that how much a lower b increases private-sector search (and other decisions) depends on how the extra search will reduce next period unemployment. This makes the Markov government differ significantly from a government in a dynamic game setting. In that case, each successive government manipulates the next government to set a lower b than it chooses. Such a strategy leads to high consumption (high current b) and low future unemployment (low future b and hence high search).

2.3.3 The Role of Commitment

By comparing first-order conditions of the Ramsey government (RAM) and the Markov government (MAR), two key differences emerge.

First, the Markov optimality conditions do not contain promised marginal values from previous period (μ_{t-1} and γ_{t-1}) as the Ramsey conditions do, because the Markov government lacks commitment to future policies. μ_{t-1} and γ_{t-1} are the marginal values to the unemployed workers and firms in period $t-1$, respectively. These marginal values are affected by expected policy and allocations in period t. For example, higher benefits in period t reduce expected gain from search and vacancy posting thus decreasing search and vacancy posting in period $t-1$. Because the Markov government cannot commit, it does not internalize how current policy affects incentives in the previous period. As a result, its policy does not depend on the values of μ_{t-1} and γ_{t-1}. In contrast, the Ramsey government chooses policies that can deliver these promises, thus their presence in the Ramsey optimality conditions.

The second difference is the presence of policy derivatives in the Markov auxiliary functions. In particular,

$$\begin{aligned}
\eta_{1u'} &\equiv \frac{\partial \eta_1}{\partial u'} + \underbrace{\frac{\partial \eta_1}{\partial b'}\Psi'_u + \frac{\partial \eta_1}{\partial w'}W'_u + \frac{\partial \eta_1}{\partial s'}S'_u + \frac{\partial \eta_1}{\partial \theta'}\Theta'_u}_{\text{disciplining effect}} \\
\eta_{2u'} &\equiv \underbrace{\frac{\partial \eta_2}{\partial w'}W'_u + \frac{\partial \eta_2}{\partial \theta'}\Theta'_u}_{\text{disciplining effect}}
\end{aligned}$$

The Markov government chooses current policy, knowing how the the private sector will behave given the policy. [12] The government correctly anticipates how future policy will depend on current policy via the state of the economy. Thus, future policies are functions of the states variables. Current Markov government can induce the future governments to choose certain policy by changing the states of the economy. More specifically, the Markov government, by choosing current benefit level, affects search intensity in the current period, [13] thus affecting next period's unemployment (and next period's state) holding other things constant. This disciplining effect is captured by the presence of policy derivatives in the auxiliary functions. In contrast, because the Ramsey government can commit to future policies, its policy is a sequence of states-contingent plan. In particular, the Ramsey government chooses in period 0 a sequence of state-contingent policies and allocations for all future periods.

2.3.4 The Case of Separable Preference

To illustrate the role of disciplining effect in the determination of benefit, we consider a special case of the Markov equilibrium. With preference separable in consumption and

[12] Equivalently, in the alternative setup here, the government chooses both policy and private-sector allocations, taking into consideration the private-sector optimality conditions.

[13] The effect of benefit level on current period search works through non-separability of preference. Under our parametrization, lower benefit level increases current search intensity.

search, the Markov government's optimality condition reduces to

$$R_b + \underbrace{\left(-\frac{\eta_{3b}}{\eta_{3w}}\right)}_{=\partial w/\partial b|_{\eta_3=0}} R_w = 0 \tag{2.20}$$

Notice that this condition does not contain policy derivative. With separable preference, current benefit policy does not affect current period search and vacancy posting. As a result, the Markov government has no disciplining effect over future governments, and each government chooses benefit policy to maximize current period government objective function ($dR/db = 0$). Effectively, the Markov benefit policy equates the marginal utility of worker and unemployed (the first term in (2.20)), taking into account how benefit affects equilibrium wage (the second term). In a way, each successive government behaves like "the last emperor".

Under the assumption of fixed wages (so the wage bargaining competitive equilibrium condition disappears), the Markov equilibrium with separable preference has an analytical solution.

Proposition 1. Under the assumptions of separable utility and fixed wages $\bar{w}$, a Markov perfect equilibrium is given by

$$b = \bar{w} - h, \quad s = 0, \quad u = 1, \quad \theta = q^{-1}\left(\frac{1-\beta(1-\delta)}{\beta(\bar{z}-\bar{w})}\kappa\right).$$

The proof is straightforward. With fixed wage, (2.20) reduces to $R_b = 0$, or equivalently,

$$U_c(h + b - \tau) = U_c(\bar{w} - \tau)$$

which, given strict monotonicity of preference in consumption, entails $h + b = \bar{w}$. Since unemployment gets the same consumption as employment, it follows that $s = 0$ and $u = 1$. When wages are fixed, steady-state market tightness is also fixed. Later, we will show that the Markov equilibria with non-separable utility converge smoothly to this equilibrium steady-state.

2.4 Calibration

We describe our calibration strategy in this section. The model period is taken to be one week. We calibrate the Markov equilibrium to match important features of the U.S. labor market.

The utility function is

$$U(c,s) = \frac{1}{1-\sigma}\left(\left[\frac{c}{v(s)}\right]^{1-\sigma} - 1\right)$$

where $v(s)$ is the cost of search. For $\sigma \neq 1$, this utility function represents a preference non-separable in c and s. We assume $v(\cdot)$ is a non-negative, strictly increasing and convex function, with the property that $v(0)$ is bounded and $v(0) > 0$. We choose the search cost function to be

$$v(s) = \exp\left(\frac{A}{1+\phi}[(1-s)^{-(1+\phi)} - 1] - (A-1)s\right)$$

This functional form is chosen to guarantee that s is strictly less than 1. In particular, for any $A > 0$, v exhibits positive and increasing marginal cost, $v_s(s) > 0$ and $v_{ss}(s) > 0$, $v(1) = v_s(1) = \infty$, and $v(0) = v_s(0) = 1 > 0$. With this functional form for search cost, when $\sigma = 1$, the utility function is reduced to $\log c - \log v(s)$, which is utility function often used in the literature. When $\sigma \neq 1$, the utility function features non-separability between consumption and search. This allows the Markov government to have disciplining power over its successor.

We adopt the matching function from den Haan, Ramey, and Watson (2000) which is also used in Hagedorn and Manovskii (2008),

$$M(su, v) = \frac{(su)v}{[(su)^{\chi} + v^{\chi}]^{1/\chi}}$$

Together with the search cost function, this matching function guarantees that both the job-finding rate $f(\theta)s$ and the job-filling rate $q(\theta)$ are always strictly less than 1.

As in Shimer (2005), labor productivity z_t is taken to be average real output per person in the non-farm business sector. This measure is taken from the seasonally adjusted quarterly data constructed by the Bureau of Labor Statistics. We normalize the mean productivity to be $\bar{z} = 1$, and assume the shock to z follows an AR(1) process:

$$\log z_t = \rho \log z_{t-1} + \sigma_\epsilon \epsilon_t$$

Table 2.1: Summary of Calibration

Parameter	Description	Value
β	Discount factor	$0.99^{1/12}$
σ	Coefficient of relative risk aversion	0.75
h	Value of non-market activity	0.4
A	Disutility of search	0.024
ϕ	Search cost curvature	1
ζ	Bargaining weight	0.5
δ	Separation rate	0.008
χ	Matching parameter	0.644
κ	Vacancy posting cost	0.58
ρ	Persistence of productivity	0.9895
σ_ϵ	Std of innovation to productivity	0.0034

where $\rho \in [0,1)$, $\sigma_\epsilon > 0$, and ϵ_t are i.i.d. standard normal random variables. The parameters are estimated to be $\rho = 0.9895$ and $\sigma_\epsilon = 0.0034$ at the weekly frequency.

We set the discount factor $\beta = 0.99^{1/12}$, giving a quarterly discount factor of 0.99. The coefficient of relative risk aversion is $\sigma = 0.75$. Although this number is small compared to those used in the macro literature, we believe it is within reasonable range given the weekly frequency. [14] Hagedorn and Manovskii (2008) estimate weekly job separation rate to be 0.0081. They also estimate the costs of vacancy creation to be 58% of weekly labor productivity. Therefore, we set the job separation parameter $\delta = 0.0081$ and cost of vacancy posting $\kappa = 0.58$.

The value of non-market activity is taken to be $h = 0.4$, a common value used in the search literature. [15] The worker's share in wage bargaining is set at 0.5, so that worker and firm have equal share of bargaining power. [16] We set the search cost curvature parameter ϕ to 1 in the benchmark model, and investigate the sensitivity of main results under different values of ϕ. [17]

[14] Hopenhayn and Nicolini (1997) also set the relative risk aversion coefficient in the range of $0.5-0.75$ for a weekly frequency.

[15] Shimer (2005) uses a value of h to be 40% of wage. Hall (2006) estimates a value of leisure relative to productivity at about 43%.

[16] The literature uses a wide range of bargaining power parameter. Shimer (2005) uses a higher bargaining power parameter of 0.72. Hagedorn and Manovskii (2008) use elasticity of wage to productivity to estimate bargaining power share at 0.052. Nakajima (2012b) estimates a bargaining share of 0.07 in a setup with elastic labor supply.

[17] Sensitivity analysis is in Appendix B.2.

Table 2.2: Summary Statistics

Statistic		z	u'	v	v/u
Quarterly U.S. data 1951-2004					
standard deviation		0.013	0.125	0.139	0.259
correlation matrix	z	1	-0.302	0.460	0.393
	u'	-	1	-0.919	-0.977
	v	-	-	1	0.982
	v/u	-	-	-	1
Calibrated Markov economy					
standard deviation		0.013	0.005	0.009	0.013
correlation matrix	z	1	-0.954	0.974	0.999
	u'	-	1	-0.861	-0.951
	v	-	-	1	0.976
	v/u	-	-	-	1

We jointly calibrate (1) the matching function parameter χ, and (2) the level parameter of search cost A, to match the mean job-finding rate and mean job-filling rate. These data targets are directly measured in the U.S. data from 1951-2004. Shimer (2005) reports a monthly job-finding rate of 0.45 and job-filling rate of 0.71. We convert them to weekly job-finding rate of 0.139 and job-filling rate of 0.266. [18] The calibration results are summarized in Table 2.1.

Table 2.2 compares some labor market statistics in the U.S. economy and the calibrated Markov economy. [19] The calibrated model does a good job generating the relevant correlations. In particular, the model delivers negative correlation between unemployment and vacancy, thus preserving the Beveridge-curve relationship. Because we target only first moments, the calibrated model generates much lower volatility than the U.S. economy. We address this issue by introducing wage rigidity in later section.

[18] We use a derivative-free algorithm for least-squares minimization to perform joint calibration. See Zhang, Conn, and Scheinberg (2010) for details.

[19] Standard deviations and correlations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

Table 2.3: Steady States

Statistic	Ramsey	Markov
benefit, b	0.318	0.564
wages, w	0.976	0.981
search, s	0.690	0.329
vacancy, v	0.036	0.028
unemployment, u	0.023	0.054
replacement ratio(%)	32.6	57.5
average consumption	0.963	0.950
consumption equivalent welfare change(%)	–	1.52

Note: Steady states are computed using parameters calibrated to Markov equilibrium.

2.5 Quantitative Analysis

In this section, we describe the quantitative results of the model. In order to investigate how the economy behaves under the Ramsey policy and the Markov policy, we bring the calibrated parameters into the model under each policy.

2.5.1 Steady State Comparison

Table 2.3 compares steady states under the Ramsey and Markov policies. Not surprisingly, the Ramsey economy performs better than the Markov economy. The Markov government gives a much higher unemployment benefit than does the Ramsey government. The replacement ratio is 58% in the Markov economy, as opposed to 33% in the Ramsey economy. Because we use the non-commitment economy as calibration target, the replacement ratio in the Markov economy is much closer to the 60% replacement ratio found in the U.S. economy. With higher benefit level, unemployed workers have less incentives to search. In fact, the Markov economy has much lower search intensity than the Ramsey economy.

Higher benefits give workers higher outside options, so wages are slightly higher in the Markov economy. Higher wages indicate lower profits for firms, and hence lower vacancy posting under the Markov policy. In addition, lower search as a result of higher benefits also means lower per-vacancy filling-rate probability for firms, and hence even lower vacancy posting. Lower search and lower vacancy posting lead to a much higher unemployment level in the Markov economy. Therefore, output is lower and agents consume less in the Markov economy. In terms of welfare, the average consumption in the Markov economy has to increase by 1.52% to be equivalent to the Ramsey economy in the steady state.

Table 2.3 highlights the importance of commitment. The government has two opposing incentives. One is insurance, through providing higher unemployment benefit to help workers smooth consumption. The other incentive is job creation, by giving lower benefits to encourage search and vacancy posting, thereby lowering unemployment and increasing output. Ideally, the government would like higher benefit in the current period and lower benefits in the following periods because, as discussed before, current search and job posting are mainly affected by expectations of future benefits. But when the government lacks the ability to commit to future policies – as in the case of the Markov government and almost all governments in reality – it cannot make any credible promise of low benefit in the future. As a result, such government consistently provides higher than optimal benefits and leads the economy into a state of high unemployment, low output and low welfare.

2.5.2 Policy Functions

In this section, we present and compare Ramsey and Markov equilibrium policy functions solved using cubic spline projection method.

Figure 2.3 plots the Ramsey policy (left) and the Markov policy (right) functions for unemployment benefit (top panels) and next period unemployment (bottom panels), holding productivity at the steady state level. [20] In each plot, the solid line represents policy function, and the dashed line indicates steady state unemployment rate. [21]

First, consider unemployment benefit (the top panels). The optimal unemployment benefit in the Ramsey case is decreasing in unemployment level, whereas the Markov benefit is increasing in unemployment. One key difference between these two governments is that the Ramsey government internalizes the impact of its current policy on

[20] The Ramsey policy function plots also hold promised marginal utilities μ_- and γ_- at their respective steady state level. Note that even though we solve Ramsey policies as functions, the solution to a Ramsey problem really should be understood as sequences of variables from $t = 0$ to $t = \infty$, given some initial state, (z_0, u_0) in this case.

[21] Appendix B.3 contains other policy function plots, holding either unemployment or productivity at steady state.

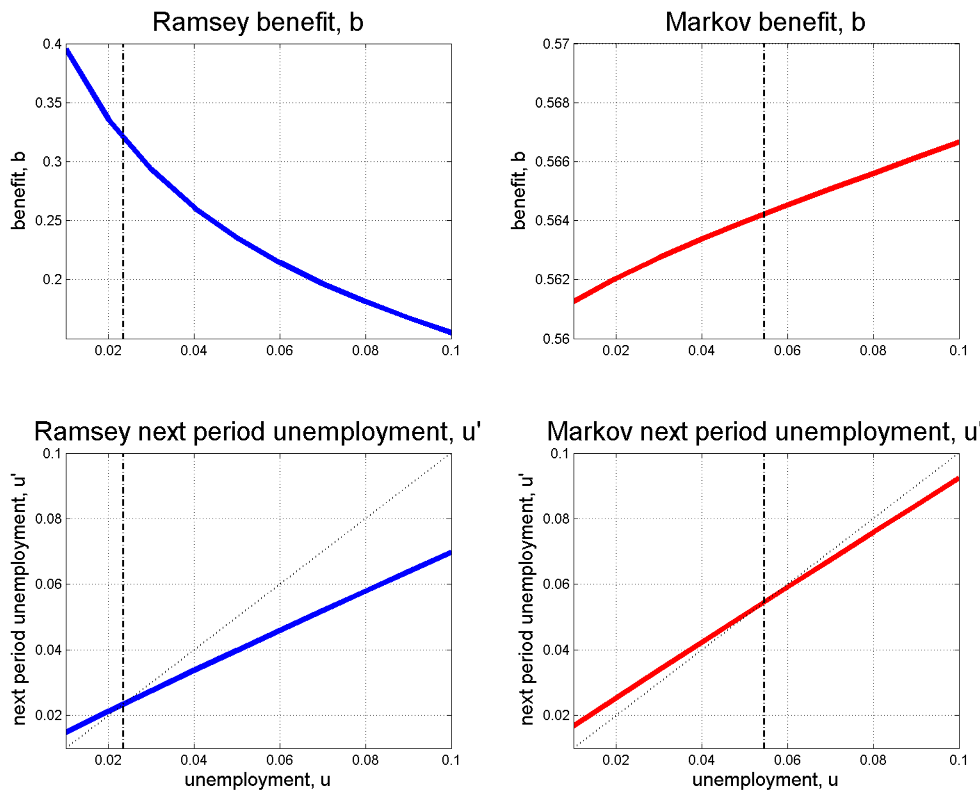

Figure 2.3: Ramsey (left) and Markov (right) benefit (top panels) and unemployment (bottom panels) policy functions holding productivity at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady-state unemployment level. The bottom panels also plot the 45° line (the thin dotted).

the actions of private sector in previous periods. When unemployment level is high, the marginal social benefit of job creation is higher, because the expected output gain of increasing vacancy posting is proportional to the number of unemployed workers. Thus the Ramsey government reduces unemployment benefit when unemployment is high, in order to induce more search and vacancy posting in the previous period.

In contrast, the Markov government considers the previous period foregone and hence does not internalize how previous period's expectation of current policy impacts the economy in the past. At the same time, as more workers are unemployed, the Markov government, with a utilitarian objective function, has a stronger motive to provide insurance and help smooth consumption. So the Markov government increases unemployment benefits by redistributing more from the employed to the unemployed. Although the government also has an incentive to encourage search, its inability to commit means the government keeps postponing the action to the next period.

The bottom panels of Figure 2.3 plot the next period unemployment policy functions u' associated with the Ramsey policy (left) and the Markov policy (right). In both cases, the policy function is increasing in current unemployment and coincides with the 45-degree line once at the steady state. Notice that the slope of the Ramsey unemployment is flatter than that of the Markov unemployment. This is because the Ramsey government, by planning a sequence of policies at time 0, has more control over the economy, and thus can move the next period unemployment further away from current unemployment. The Markov government, in contrast, can only influence the next period economy through the disciplining effect on the next government, and thus has smaller power over the state of the economy.

Figure 2.4 plots the Ramsey (left) and the Markov (right) benefit policy functions, holding unemployment at the steady state level. The Ramsey and Markov unemployment benefits are decreasing and increasing, respectively, in productivity. In other words, when productivity is low, optimal benefit is high whereas the Markov government provides low benefit. The difference comes again from the lack of commitment by the Markov government. From the perspective of the Ramsey government, the marginal social benefit of job creation is lower when productivity is low, since the output of each firm-worker pair is low. As a result, the marginal social cost – in the form of lower search and fewer vacancy postings – of unemployment benefit is low. So the Ramsey government provides high benefit.

In contrast, the Markov government does not internalize the changing social marginal cost of benefits in the form of job creation. The Markov government weighs the welfare gain from redistribution against the financing cost of benefits. When productivity is low, output and the aggregate resource in the economy are low. As a result, the marginal cost of financing benefits is high, and so the Markov government provides low benefits. In addition, with persistent shocks, low productivity implies that future productivities are also likely to be low. With expectations of low future productivity, firms reduce vacancy posting. The Markov government reduces unemployment benefit to encourage current

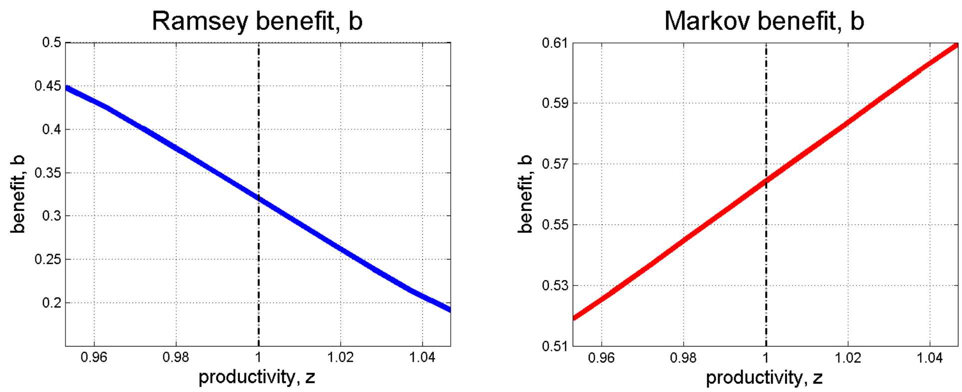

Figure 2.4: Ramsey (left) and Markov (right) benefit policy functions holding unemployment at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady-state productivity level.

period search and vacancy posting, [22] thus increasing future aggregate resources.

2.5.3 Continuity of Markov Perfect Equilibrium

As in the previous literature on dynamic games, we cannot prove general existence or uniqueness results for the Markov perfect equilibrium. But with fixed wages, we can show the continuity of Markov equilibrium policy rules. Proposition 1 provides the analytical solution for Markov equilibrium under the assumptions of separable utility ($\sigma = 1$) and fixed wages. We set wages fixed at the flexible-wage Markov equilibrium steady-state level $\bar{w} = 0.981$.

Figure 2.5 shows that the Markov equilibrium converges smoothly to the equilibrium with separable preference as $\sigma \to 1$. The figure plots the Markov equilibrium steady-state benefit (left) and unemployment (right) for economies with relative risk aversion σ ranging from 0.6 to 1, holding all other parameters as given in Table 2.1. Circles indicate the 25 values of σ for which the Markov equilibrium is computed numerically. The values for $\sigma = 1$ correspond to the equilibrium computed analytically in Proposition 1. At $\sigma = 1$, the equilibrium features high benefit and high unemployment. As σ increases toward 1, both benefit and unemployment rise.

[22] This effect works because preferences are non-separable in consumption and search intensity. Under our parameterization, the cross derivative of utility in benefit and search is negative. So when benefits are low, the marginal utility (cost) of search is high (low), and thus search is high. Higher search intensity increases the per-vacancy job-filling rate, so firms have more incentive to post vacancy.

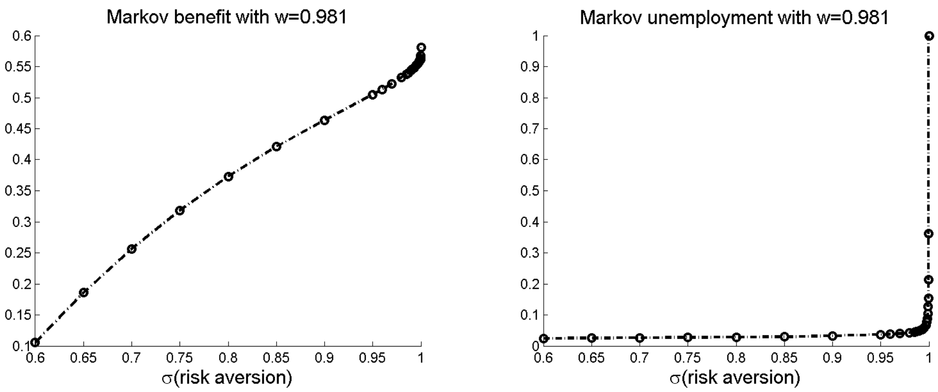

Figure 2.5: Continuity of Markov-perfect equilibrium. Markov equilibrium steady-state benefit (left) and unemployment (right) for economies with relative risk aversion σ ranging from 0.6 to 1. Wages are fixed at $\bar{w} = 0.981$. All other parameters follow Table 2.1. Circles indicate the 25 values of σ for which the Markov equilibrium is computed numerically. The values for $\sigma = 1$ correspond to the equilibrium computed analytically in Proposition 1.

2.5.4 Dynamics

To understand how the Ramsey and Markov economy behave over time, we simulate each economy. Figure 2.6 plots and compares the dynamic responses of key variables in the Ramsey and Markov economy to a 1% drop in productivity. [23] The optimal benefit level initially jumps up, then falls for about 30 weeks following the shock, and slowly reverts to its pre-shock level. Unemployment rises in response to the drop in productivity and continues rising for about 10 weeks before falling back to its pre-shock level. The Markov government, however, reduces benefits in response to lower productivity, and slowly raises benefits back to its pre-shock level. Because of the different initial responses in benefit policy, unemployment in the Markov economy also responds markedly differently compared to the Ramsey economy. Unemployment jumps up only slightly when the shock hits, then rises for 15 weeks, and slowly falls back to the pre-shock level.

Such responses to a negative shock are consistent with the properties of the Ramsey and Markov policy functions. Immediately after the negative shock, productivity is low, and so is the social value of employment. As a result, the Ramsey government tolerates the rise in unemployment. The Markov government, not internalizing

[23] Appendix B.3 contains plots of dynamic responses for other labor market variables.

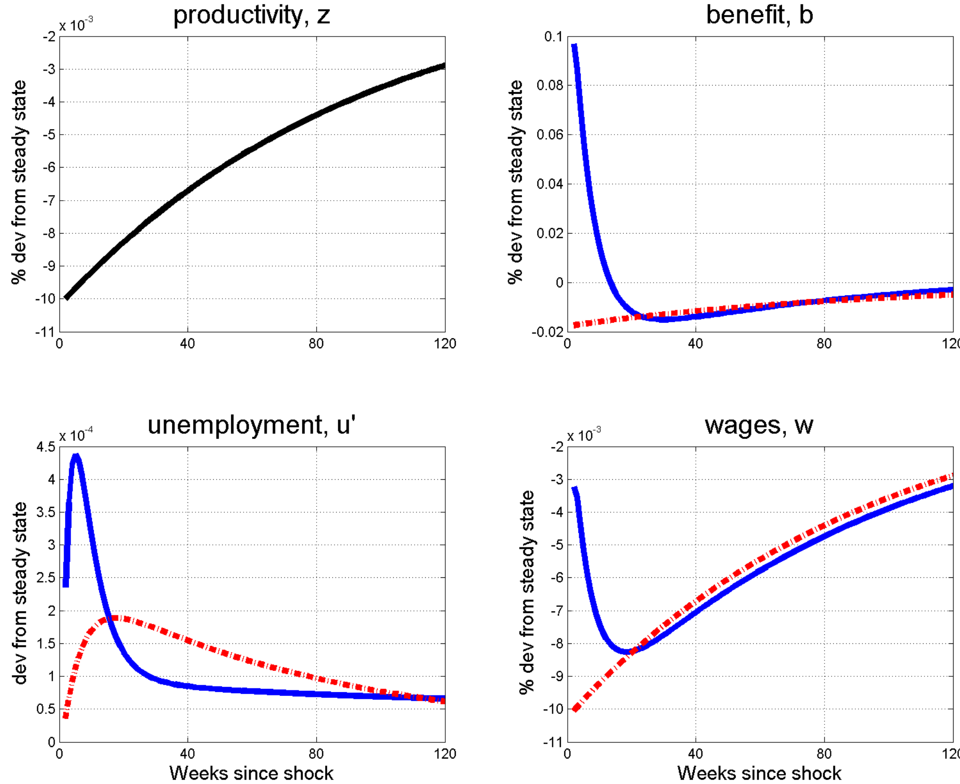

Figure 2.6: Ramsey (solid line) and Markov (dashed line) responses to a 1% drop in productivity.

the changing social value of job creation, lowers benefits in response to lower aggregate output after the shock. As unemployment rises, the social benefit of creating more vacancies increases relative to the benefit of providing insurance, and the Ramsey government therefore cuts unemployment benefits to reduce unemployment. The Markov government, however, does not internalize the effect of benefits on job creation in the previous period; instead, the Markov benefit rises as unemployment rate increases and productivity rises, as more incentives to redistribute and more resources to redistribute respectively. Because the Markov government does not increase benefits immediately after the shock, unemployment does not peak until 15 weeks after the shock, and peaks at a lower level, in deviation terms, than unemployment in the Ramsey economy. But because the Markov government keeps raising benefit following the shock, the Markov

Table 2.4: Simulated Statistics under Ramsey and Markov Policy

Statstic	productivity	benefit	unemployment	wages	search	vacancy
Ramsey policy						
mean	1	0.319	0.023	0.976	0.689	0.036
std. dev.	0.013	0.055	0.013	0.012	0.007	0.013
Markov policy						
mean	1	0.564	0.054	0.980	0.329	0.028
std. dev.	0.013	0.023	0.005	0.013	0.000	0.013

Note: Means are reported in levels. Standard deviations are reported in log quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

economy features a much slower recovery in unemployment.

Turning to wages. Wages fall less, in percent deviation terms, in the Ramsey economy than they do in the Markov economy. This is because the initial rise in Ramsey benefits smooths the fall in wages through an increase in the worker's outside option. Wages also fall for a longer period – for about 15 weeks before picking up – under the optimal policy, whereas wages in the Markov economy dip upon impact, and rise monotonically back to their pre-shock level.

Table 2.4 reports the moments of key variables when the model is simulated under the optimal policy and the Markov policy. Compared to the optimal policy, search intensity under the Markov policy has minimal volatility (rounded to zero). This is because the Markov policy, unlike the Ramsey policy, is not designed to increase search incentive when the economy is in recovery. The low search volatility contributes to lower volatility of unemployment in the Markov economy. These results show that although the optimal policy creates faster recovery, the Markov economy has less cyclical fluctuations in unemployment. The higher unemployment volatility under the optimal policy reflects the fact that the Ramsey government aims to reduce volatility of average consumption (and welfare), at the cost of higher unemployment fluctuation. In fact, consumption (welfare) volatility in the Ramsey economy is 12.36% (12.34%), lower than that of 13.68% (13.58%) under the Markov government.

Vacancy posting, on the other hand, has the same volatility under both policies, because vacancy posting is mainly driven by expected future aggregate productivity.

Table 2.5: Summary Statistics under Wage Rigidity

Statistic		z	u'	v	v/u
Quarterly U.S. data 1951-2004					
standard deviation		0.013	0.125	0.139	0.259
correlation matrix	z	1	-0.302	0.460	0.393
	u'	-	1	-0.919	-0.977
	v	-	-	1	0.982
	v/u	-	-	-	1
Calibrated Markov economy					
standard deviation		0.013	0.125	0.254	0.360
correlation matrix	z	1	-0.938	0.976	0.999
	u'	-	1	-0.847	-0.934
	v	-	-	1	0.981
	v/u	-	-	-	1

Since the two economies follow the same productivity process and experience the same shocks, the volatility of vacancy posting – and wages – do not differ much between the two economies.

Wage Rigidity

It is well-known that without wage rigidity the search and matching model cannot easily generate the scale of cyclical fluctuations observed in reality. [24] This is why the second-moments in Table 2.4 are much lower than empirical moments. To see whether the behavior of the Ramsey and Markov economies change when the economy has more realistic volatility, we introduce wage rigidity in the form of countercyclical bargaining power of workers. Specifically, we specify that the worker's bargaining power follows, [25]

$$\zeta_t = \exp(\log \bar{\zeta} + \epsilon_\zeta \log z_t)$$

[24] See, for example, Shimer (2005), Hall (2005), and Hagedorn and Manovskii (2008) for more details on wage rigidity.

[25] Similar assumptions are common in the literature. Landais, Michaillat, and Saez (2010) and Nakajima (2012a) both directly specify $w_t = \exp(\log \bar{w} + \epsilon_w z_t)$. Jung and Kuester (2015) use a cyclical bargaining power structure similar to ours in their benchmark calibration.

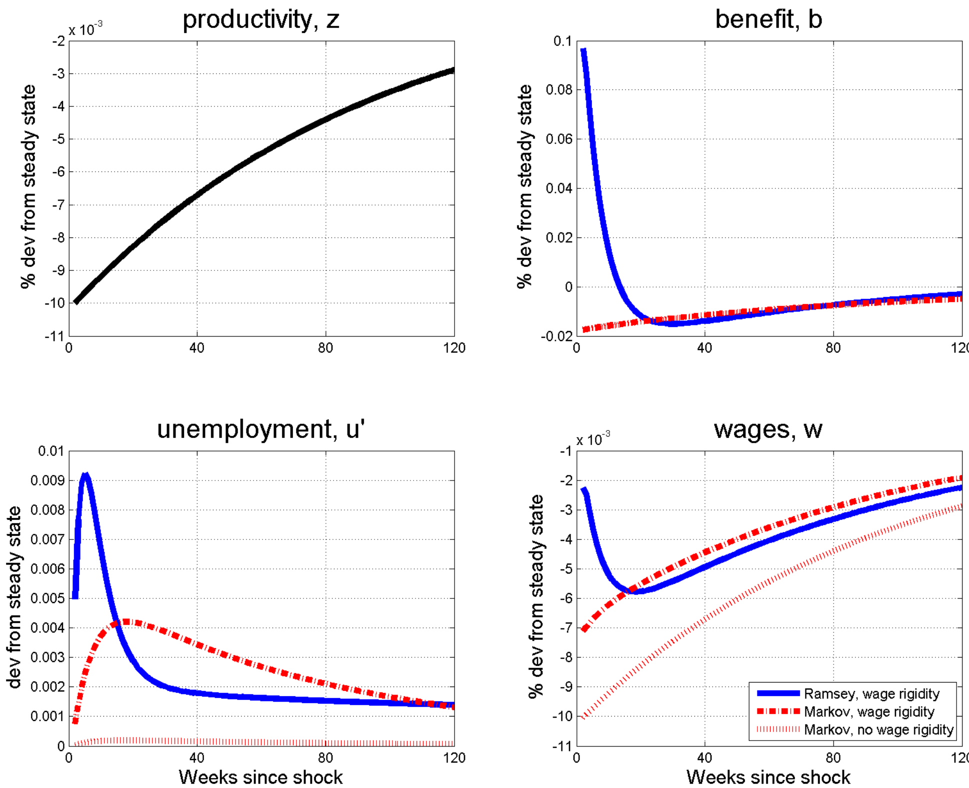

Figure 2.7: Responses to 1% drop in productivity: Ramsey (solid line) and Markov (dashed line) with wage rigidity, Markov without wage rigidity (dotted line)

where $\epsilon_\zeta < 0$ and is calibrated in the Markov economy to match the volatility of unemployment observed in the data. Hagedorn and Manovskii (2008) document quarterly standard deviation of unemployment from 1951 to 2004 to be 0.125. This implies $\epsilon_\zeta = -6.6$, so for a 1% drop in productivity, the bargaining power of workers increases by 6.6%. Table 2.5 compares labor market statistics of the U.S. economy with the Markov economy calibrated with wage rigidity. With wage rigidity, the Markov economy does a good job capturing the cyclical properties of the U.S. labor market.

Figure 2.7 plots responses of the Ramsey and Markov economies to an unexpected 1% drop in productivity with wage rigidity. Both economies behave similarly as the economies without wage rigidity. For easier comparison, Figure 2.7 also plots Markov policy responses without wage rigidity. As expected, unemployment exhibits much large cyclical fluctuations with wage rigidity, as do wages.

2.5.5 Discussions

Some limitations of our quantitative analysis should be acknowledged. First, although the model period of one week is common in the search literature, it is hard to imagine a (Markov) government making benefit decisions every week. In the U.S., benefit is rarely changed during normal times. During recessions the Congress votes on benefit policies more frequently. We believe a policy-making period of one quarter should be a reasonable choice. Second, the government in our model balances budget every period. As a result, when overall production is low, there are less resources to allocate to unemployment benefit. In practice, during recessions, the government can borrow to finance extra spending. We abstract from government debt to avoid debt-related non-commitment issue and to reduce dimensionality. Third, as in the previous literature on dynamic games, we cannot prove general existence or uniqueness results for the Markov equilibrium. Nevertheless, we use the case of fixed wages to demonstrate smooth convergence. When increasing relative risk aversion σ toward 1 – in which case with preference being separable, the Markov equilibrium reduces to a simple problem where solution is analytical and existence can be proven – the equilibrium policy rules converge smoothly to the analytical equilibrium.

2.6 Conclusion

This paper studies how a benevolent government should trade off the benefits and costs of unemployment benefits, when the government can and cannot commit to its future policy. The former and the latter are the Ramsey and the Markov economy respectively. There are higher unemployment benefits and higher unemployment level in the Markov economy than in the Ramsey economy. The Ramsey policy, which is optimal, raises benefits at the beginning of a recession, and gradually decreases benefits over time. However, the Markov policy, which is time consistent, reduces benefits at the onset of a recession, and slowly increases benefits as the economy recovers. Furthermore, the Markov policy leads to a slower recovery of unemployment. Our findings thus highlight the importance of commitment when the government is designing optimal unemployment insurance policy.

An important direction for future research is to study the duration of unemployment benefits. Extending the durations also serves to provide more insurance at the costs of discouraging search. It will be interesting to see how the government changes both the level and the duration of UI benefits over the business cycle.

Another interesting direction is to allow the government and the private sector to change their behavior at a different frequency. In the paper, workers search and the government adjusts UI benefits both at a weekly frequency. It will be more realistic if the government changes its policy on a quarterly basis while workers still search at a weekly frequency.

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Appendix A

Appendix to Chapter 1

A.1 Data Description

I describe the variables and data sources used in Chapter 1.

Default Events: I consider 23 default events. The list is from Mendoza and Yue (2012): Argentina (1982, 2002), Chile (1983), Croatia (1992), Dominican Republic (1993), Ecuador (1999), Indonesia (1998), Mexico (1982), Moldova (2002), Nigeria (1983, 1986), Pakistan (1998), Peru (1983), Philippines (1983), Russia (1998), South Africa (1985, 1993), Thailand (1998), Ukraine (1998), Uruguay (1990, 2003), Venezuela (1995, 1998).

Real GDP: Gross domestic product at constant local currency unit. Data are annual, seasonally adjusted, 1960-2013. The source is World Bank's World Development Indicators.

Real Consumption: Household final consumption expenditure at constant local currency unit. Data are annual, seasonally adjusted, 1960-2013. The source is World Bank's World Development Indicators.

Real Investment: Gross capital formation at constant local currency unit. Data are annual, seasonally adjusted, 1960-2013. The source is World Bank's World Development Indicators.

Employment: Number of persons employed. Data are annual, seasonally adjusted, 1960-2013. The source is the Conference Board Total Economy Database.

Worked Hours: Average number of hours worked per year per worker. Data are annual, seasonally adjusted, 1960-2013. The source is the Conference Board Total Economy Database.

Debt/GDP: Total gross central or general government debt to GDP ratio. Data are annual. Time period varies for each country. The source is Reinhart and Rogoff (2011a).

A.2 Derivation of the Collateral Constraint

Figure A.1 illustrates the renegotiation game between the firm and the lender. The decision for the firm to repay or default arises after the realization of revenues but before repaying the intraperiod loan. If the firm decides to repay, the lender gets back the intraperiod loan ℓ and the firm continues its operation. At this moment, the value of the firm is $Em'W'$, where m' is the stochastic discount factor and W' is the next-period value of the firm.

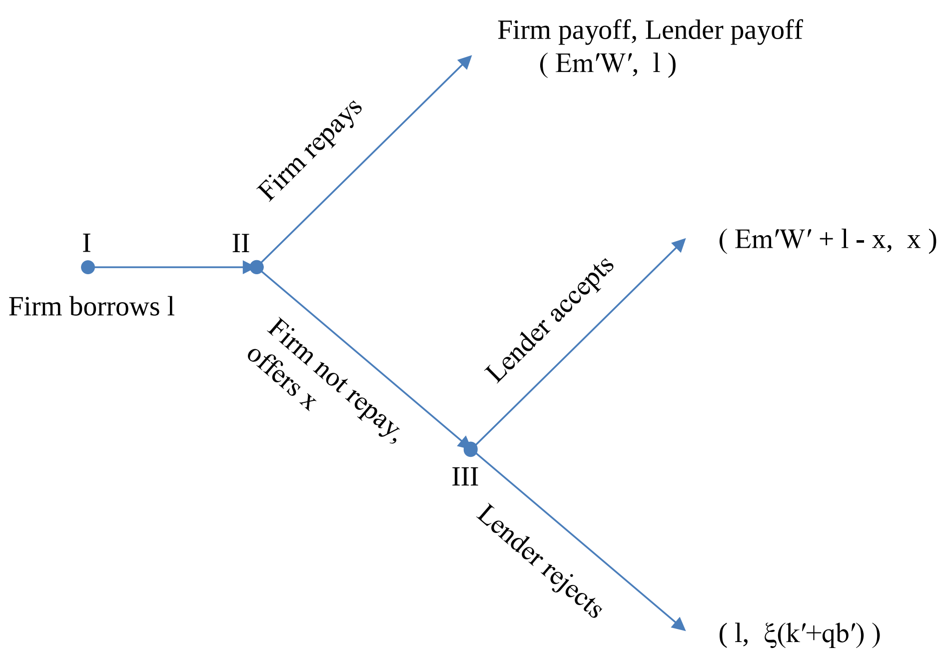

Figure A.1: Renegotiation Game

If the firm decides to not repay, the lender acquires the right to liquidate the firm's assets. Suppose the lender can acquire a fraction ξ of the firm's assets $k' + qb'$. (An equivalent way is to assume the lender is able to recover the whole value with probability ξ). However, the firm can renegotiate with the lender by offering the lender an amount of x. If the lender accepts this offer, the lender will not liquidate the firm. The firm keeps the intraperiod loan and keeps its operation, therefore its payoff is $Em'W' + \ell - x$ at this moment. But if the lender rejects the offer, the firm is liquidated and has only ℓ in hand, while the lender gets the liquidation value of $\xi(k' + qb')$.

Since it is in the interest of the firm to continue its operation, the firm has to make an offer that leaves the lender indifferent between liquidating and keeping the firm in operation. This requires the firm to offer $x \geq \xi(k' + qb')$ in order for the lender to accept.

Hence, if the intraperiod loan is more than the liquidation value of the firm, i.e. $\ell > \xi(k' + qb')$, the firm will choose not to repay and instead offer $\xi(k' + qb')$ to the lender. Anticipating this behavior, the lender will only lend

$$\ell \leq \xi(k' + qb'),$$

which gives the collateral constraint in the model.

A.3 Disaggregated Model of Banks and Firms

In this section, I describe a model where banks and firms are separate entities, and show that this model is equivalent to the baseline model where banks and firms are aggregated together into one entity.

The modeling of the government is the same as before, and I assume the government does not default in this section. The private sector now consists of households, firms and banks. Households own both firms and banks. Households supply labor to firms, receive dividends from banks and profits from firms, and put intraperiod deposits into banks.

Firms rent capital k from banks and hire labor n from households. To guarantee payments for capital and labor, firms have to take intraperiod working capital loans ℓ^f from banks. After obtaining the loan, firms use this loan to rent capital and hire labor to produce. After production is realized, firms use proceeds from output to repay the loan. I assume firms always repay this loan. The firm's problem is

$$\begin{aligned} \max_{k,n,\ell^f} \quad & F(z,k,n) - Rk - wn + \ell^f - \ell^f \\ \text{subject to} \quad & \\ \ell^f \quad & \geq \quad Rk + wn \end{aligned}$$

where R is the rental rate of capital and w is the wage rate of workers. Due to Cobb-Douglas technology, firms' profits are zero.

Banks start off a period with capital stock k and bond holdings b. In order to make working capital loan ℓ^f to firms, banks take intraperiod deposits ℓ^h from households. As in the baseline model, there is limited enforcement, where banks can choose not to repay households. This gives rise to the collateral constraint $\xi(k' + qb') \geq \ell^h$, where this constraint is derived using the same game tree as in Appendix A.2. The bank's problem is

$$\begin{aligned} W(k,b;S) &= \max_{d,k',b'} d + \beta \mathbb{E}\left[\frac{U_c(c',n')}{U_c(c,n)} W(k',b';S')\right] \\ \text{subject to} & \\ d + k' + qb' &= (1-\delta)k + Rk + b + (\ell^h - \ell^h) + (\ell^f - \ell^f) \\ \ell^h &= \ell^f \\ \xi(k' + qb') &\geq \ell^h \end{aligned}$$

If we combine the firm and the bank into one single organization, it is easy to see that this organization solves the same problem as the integrated "firm" solves in the baseline model. An alternative way to interpret is that the bank's lending to firms is limited by the assets of the bank.

A.4 First-Order Conditions for the Firm's Problem

For convenience, we reproduce the firm's problem here.

$$\begin{aligned} W(k,b;S) &= \max_{d,n,k',b'} d + \beta \mathbb{E}\left[\frac{U_c(c',n')}{U_c(c,n)} W(k',b';S')\right] \\ \text{subject to} & \\ d + k' + qb' &= (1-\delta)k + F(z,k,n) - wn + (1 - D\lambda)b \\ \xi(k' + qb') &\geq F(z,k,n). \end{aligned}$$

Let θ and μ be the Lagrange multipliers on the budget constraint and the collateral constraint respectively. The first-order conditions with respect to d, n, k' and b' are respectively

$$\begin{aligned}
1-\theta &= 0,\\
\theta(F_n-w)-\mu F_n &= 0,\\
\theta-\xi\mu &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}W_{k'}\right),\\
(\theta-\xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}W_{b'}\right).
\end{aligned}$$

The envelope conditions are

$$\begin{aligned}
W_k &= \theta(1-\delta+F_k)-\mu F_k,\\
W_b &= \theta(1-D\lambda).
\end{aligned}$$

Combining the first-order conditions and the envelope conditions, we get

$$\begin{aligned}
F_n(z,k,n) &= \frac{w}{1-\mu},\\
1-\xi\mu &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-\delta+(1-\mu')F_k(z',k',n')\right]\right),\\
(1-\xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-D'\lambda\right]\right).
\end{aligned}$$

A.5 Algorithm

The numerical solution of the model is based on a global solution that uses projection method. It consists of value function iteration for the government problem and policy function iteration for the agents. As the collateral constraint is not always binding, I also need to check for occasionally binding constraints.

The model solves for the optimal time-consistent fiscal policy, [1] so the solution searches for the fixed point where neither the government nor the agents have incentives to deviate from their policies. The idea is that given the current and future governments' policies, agents in the economy adjust their expectations and make their decisions.

[1] Klein, Krusell, and Rios-Rull (2008) solves for a time-consistent tax policy in a deterministic environment with one asset.

In turn, based on agents' decisions and the future government's policies, the current government chooses its policies to maximize agents' welfare. Government's policies are time-consistent if the current and future governments' policy functions coincide. For ease of elaboration, I denote the conditional expectations by E^k and E^b where

$$\begin{aligned} E^b(z',k',b') &= \mathbb{E}\left(U'(c')\left[1-D'\lambda\right]\right) \\ E^k(z',k',b') &= \mathbb{E}\left(U'(c')\left[1-\delta+(1-\mu')\alpha z' k'^{\alpha-1} n'^{1-\alpha}\right]\right) \end{aligned}$$

and rewrite the government value function when it does not default,

$$\begin{aligned} V^r(z,k,b) &= \max_{c,n,d,k',b',\tau,w,q,\mu} U(c,n)+\beta\mathbb{E}\left[V(z',k',b')\right] \\ &\text{subject to} \\ c &= (1-\tau)wn+d \\ d+k'+qb' &= (1-\delta)k+zk^{\alpha}n^{1-\alpha}-wn+b \\ gzk^{\alpha}n^{1-\alpha}+b &= qb'+\tau wn \\ \frac{U_n}{U_c} &= -(1-\tau)w \\ (1-\alpha)zk^{\alpha}n^{-\alpha} &= \frac{w}{1-\mu} \\ (1-\xi\mu)qU_c &= \beta E^b(z',k',b') \\ (1-\xi\mu)U_c &= \beta E^k(z',k',b') \\ \xi(k'+qb') &\geq zk^{\alpha}n^{1-\alpha}, \mu\geq 0, \text{ and } \mu[\xi(k'+qb')-zk^{\alpha}n^{1-\alpha}]=0 \end{aligned}$$

The government's value function of defaulting is similar.

The algorithm contains two loops: the inner loop iterates on the government's value functions while the outer loop iterates on the agents' expectations. The details of the algorithm consist of the following steps:

1. Create grids for productivity shocks, capital stock and bond holdings, $[\underline{z},\overline{z}]\times[\underline{k},\overline{k}]\times[\underline{b},\overline{b}]$.

2. Make initial guesses for V^0, $E^{k,0}$, and $E^{b,0}$.

3. Suppose we are at the i-th iteration.

4. At each grid point (z, k, b) and for each choice of b', I first assume the collateral constraint is binding and solve a system of eight equations (the eight constraints in the value function) with eight unknowns $\{c, n, d, k', \tau, w, q, \mu\}$ using a nonlinear equation solver based on a variant of the Newton's method.

5. If the multiplier μ is negative, I set it to zero, drop the collateral constraint and solve the system of seven equations and seven unknowns.

6. The set of variables $\{c^{*,r}(b'), n^{*,r}(b'), d^{*,r}(b'), k'^{*,r}(b'), \tau^{*,r}(b'), w^{*,r}(b'), q^{*,r}(b')\}$ and $\mu^{*,r}(b')$ are the economy's competitive equilibrium conditions if the government does not default and chooses b'.

7. In a similar fashion, I calculate the economy's competitive equilibrium conditions if the government defaults.

8. Given these solutions, I calculate the welfare $V^d(z, k, b)$ and $V^r(z, k, b) = \max_{b'}$ $\widehat{V}^r(z, k, b; b')$, and choose the optimal $b'^* \in$ argmax $\widehat{V}^r(z, k, b; b')$.

9. Use the results in step 8 to choose optimal default decision: $D^* = 1$ if $V^d > V^r$ and $D^* = 0$ otherwise.

10. Use the results in step 8 to update $V(z, k, b) = \max\left\{V^r(z, k, b), V^d(z, k, b)\right\}$.

11. Repeat step 8 to step 10 until the value function V converges. Denote it by V^i and the associated policies by b'^i and D^i.

12. Obtain competitive equilibrium conditions. For example, $c^i = c^{*,d}$ if $D^i = 1$ and $c^i = c^{*,r}(b'^i)$ if $D^i = 0$.

13. Use the values in step 12 to update agents' conditional expectations using the Gauss-Hermite quadrature method. Denote them by $E^{k,i}$ and $E^{b,i}$.

14. Repeat step 4 to step 13 until the expectations E^k and E^b converge.

Appendix B

Appendix to Chapter 2

B.1 Derivations

B.1.1 Derivation of Private Sector Optimality Conditions

Solving unemployed person's problem by taking derivative with respect to s

$$\frac{-U_s(h+b-\tau,s)}{f(\theta)} = \beta[V^e(z',u') - V^u(z',u')] \tag{B.1}$$

using worker's bellman equations

$$V^e(z,u) - V^u(z,u) = U(w-\tau,0) - U(h+b-\tau,s) + \beta(1-f(\theta)s-\delta)[V^e(z',u') - V^u(z',u')]$$

combining the two equations

$$V^e(z,u) - V^u(z,u) = U(w-\tau,0) - U(h+b-\tau,s) + (1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)} \tag{B.2}$$

update one period, take expectations and substitute into (B.1)

$$\begin{aligned}\frac{-U_s(h+b-\tau,s)}{f(\theta)} &= \beta\mathbb{E}\Big[U(w'-\tau',0) - U(h+b'-\tau',s') \\ &\quad +(1-f(\theta')s'-\delta)\frac{-U_s(h+b'-\tau',s')}{f(\theta')}\Big]\end{aligned}$$

From unmatched firm's value function, assuming free entry, i.e. $J^u(z,u)=0$

$$\frac{\kappa}{q(\theta)} = \beta\mathbb{E}J^e(z',u') \tag{B.3}$$

then firm's value function can be rewritten as

$$J^e(z,u) = z - w + (1-\delta)\frac{\kappa}{q(\theta)} \tag{B.4}$$

update one period, take expectations and substitute into (B.3)

$$\frac{\kappa}{q(\theta)} = \beta\mathbb{E}\left[z' - w' + (1-\delta)\frac{\kappa}{q(\theta')}\right]$$

Take first-order condition of the Nash bargaining problem (2.7) with respect to w

$$\zeta U_c(w-\tau,0)\Big[J^e(z,u) - J^u(z,u)\Big] \quad = \quad (1-\zeta)\Big[V^e(z,u) - V^u(z,u)\Big]$$

substitute in (B.2) and (B.4)

$$\begin{aligned} & \zeta U_c(w-\tau,0)\left[z - w + (1-\delta)\frac{\kappa}{q(\theta)}\right] \\ = \quad & (1-\zeta)\left[U(w-\tau,0) - U(h+b-\tau,s) + (1-f(\theta)s-\delta)\frac{-U_s(h+b-\tau,s)}{f(\theta)}\right] \end{aligned}$$

B.1.2 Definition of Auxiliary Functions in the Ramsey Problem

$$\tilde{\eta}_0(u_t,s_t,\theta_t,u_{t+1}) = u_{t+1} - \delta(1-u_t) - (1-f(\theta)s)u_t$$

$$\begin{aligned} & \tilde{\eta}_1(u_t,b_t,\tau_t,s_t,\theta_t,u_{t+1},b_{t+1},\tau_{t+1},w_{t+1},s_{t+1},\theta_{t+1}) \\ = \quad & \frac{-U_s(h+b_t-\tau_t,s_t)}{f(\theta_t)} \\ & -\beta\mathbb{E}_t\left[U(w_{t+1}-\tau_{t+1},0) - U(h+b_{t+1}-\tau_{t+1},s_{t+1})\right] \\ & +\beta\mathbb{E}_t\left[(1-f(\theta_{t+1})s_{t+1}-\delta)\frac{-U_s(h+b_{t+1}-\tau_{t+1},s_{t+1})}{f(\theta_{t+1})}\right] \end{aligned}$$

$$\tilde{\eta}_2(\theta_t,z_{t+1},w_{t+1},\theta_{t+1}) = \frac{\kappa}{q(\theta_t)} - \beta\mathbb{E}_t\left[z_{t+1} - w_{t+1} + (1-\delta)\frac{\kappa}{q(\theta_{t+1})}\right]$$

$$\begin{aligned} & \tilde{\eta}_3(z_t,u_t,b_t,\tau_t,w_t,s_t,\theta_t) \\ = \quad & \zeta U_c(w_t-\tau_t,0)\left[z_t - w_t + (1-\delta)\frac{\kappa}{q(\theta_t)}\right] \\ & -(1-\zeta)\left[U(w_t-\tau_t,0) - U(h+b_t-\tau_t,s_t) + (1-f(\theta_t)s_t-\delta)\frac{-U_s(h+b_t-\tau_t,s_t)}{f(\theta_t)}\right] \end{aligned}$$

B.1.3 Derivation of Markov GEE

Throughout this section, we drop the dependence of functions on productivity shock z to economize on notation. Combine government first-order conditions,

$$\begin{aligned}
& \frac{1}{\eta_{0s}}\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right] + \frac{\eta_{1u'}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) \\
& -\frac{\eta_{2u'}}{\eta_{2\theta}}\left[\frac{\eta_{0\theta}}{\eta_{0s}}\left(R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right) + \frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) + \frac{\eta_{3\theta}}{\eta_{3w}}R_w\right] \\
= & \ \beta\Omega'_u \qquad (FOC)
\end{aligned}$$

Rewrite Bellman equation in shorthand

$$\Omega(u) \quad = \quad R(u, \Psi(u), W(u), S(u)) + \beta\Omega(\Pi(u))$$

taking derivative of Bellman equation wrt u

$$\Omega_u \quad = \quad R_u + R_b\Psi_u + R_w W_u + R_s S_u + \beta\Omega'_u\Pi_u \qquad (ENV)$$

combine FOC and ENV to eliminate $\beta\Omega'_u$

$$\begin{aligned}
\Omega_u \quad = \quad & R_u + R_b\Psi_u + R_w W_u + R_s S_u \\
& +\Pi_u\left\{\frac{1}{\eta_{0s}}\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right] + \frac{\eta_{1u'}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right)\right\} \\
& -\Pi_u\left\{\frac{\eta_{2u'}}{\eta_{2\theta}}\left[\frac{\eta_{0\theta}}{\eta_{0s}}\left(R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right)\right.\right. \\
& \left.\left. + \frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) + \frac{\eta_{3\theta}}{\eta_{3w}}R_w\right]\right\}
\end{aligned} \tag{B.5}$$

differentiate η_1 and η_2 with respect to u

$$\begin{aligned}
0 &= \eta_{1u} + \eta_{1b}\Psi_u + \eta_{1s}S_u + \eta_{1\theta}\Theta_u + \eta_{1u'}\Pi_u \\
0 &= \eta_{2\theta}\Theta_u + \eta_{2u'}\Pi_u
\end{aligned}$$

re-arrange

$$\begin{aligned}
\frac{\eta_{1u'}}{\eta_{1b}}\Pi_u &= -\frac{\eta_{1u}}{\eta_{1b}} - \Psi_u - \frac{\eta_{1s}}{\eta_{1b}}S_u - \frac{\eta_{1\theta}}{\eta_{1b}}\Theta_u \\
\frac{\eta_{2u'}}{\eta_{2\theta}}\Pi_u &= -\Theta_u
\end{aligned}$$

substitute into (B.5)

$$\begin{aligned}
\Omega_u &= R_u + R_b\Psi_u + R_w W_u + R_s S_u \\
&+\Pi_u \frac{1}{\eta_{0s}}\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right] \\
&-\left(\frac{\eta_{1u}}{\eta_{1b}} + \Psi_u + \frac{\eta_{1s}}{\eta_{1b}}S_u + \frac{\eta_{1\theta}}{\eta_{1b}}\Theta_u\right)\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) \\
&+\Theta_u\left[\frac{\eta_{0\theta}}{\eta_{0s}}\left(R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right) + \frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) + \frac{\eta_{3\theta}}{\eta_{3w}}R_w\right] \\
&= R_u + \left[\frac{1}{\eta_{0s}}\Pi_u + S_u + \frac{f_\theta(\theta)s}{f(\theta)}\Theta_u\right]\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right)\right] - \frac{\eta_{1u}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) \\
&+\left[W_u + \frac{\eta_{3b}}{\eta_{3w}}\Psi_u + \frac{\eta_{3\theta}}{\eta_{3w}}\Theta_u - \frac{\eta_{3s}}{\eta_{3w}}\left(\frac{1}{\eta_{\eta_{0s}}}\Pi_u + \frac{f_\theta(\theta)s}{f(\theta)}\Theta_u\right)\right]R_w \qquad \text{(B.6)}
\end{aligned}$$

given the worker flow equation

$$\Pi(u) = \delta(1-u) + [1 - f(\Theta(u))S(u)]u$$

differentiate wrt u

$$\frac{1}{uf(\theta)}\Pi_u + S_u + \frac{f_\theta(\theta)s}{f(\theta)}\Theta_u = \frac{1-\delta-f(\theta)s}{uf(\theta)}$$

given $\eta_3\left[u, \Psi(u), W(u), S(u), \Theta(u)\right] = 0$, differentiate wrt u

$$W_u + \frac{\eta_{3b}}{\eta_{3w}}\Psi_u + \frac{\eta_{3\theta}}{\eta_{3w}}\Theta_u = -\frac{\eta_{3u}}{\eta_{3w}} - \frac{\eta_{3s}}{\eta_{3w}}S_u$$

substitute into (B.6)

$$\begin{aligned}
\Omega_u &= R_u - \frac{\eta_{3u}}{\eta_{3w}}R_w + \frac{1-\delta-f(\theta)s}{uf(\theta)}\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right] \\
&-\frac{\eta_{1u}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right)
\end{aligned}$$

update and substitute into FOC, we get the GEE

$$\begin{aligned}
& \underbrace{\frac{1}{uf(\theta)}\left[R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right]}_{\lambda\eta_{0u'}} + \underbrace{\frac{\eta_{1u'}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right)}_{\mu\eta_{1u'}} \\
& \underbrace{-\frac{\eta_{2u'}}{\eta_{2\theta}}\left[\frac{f_\theta(\theta)s}{f(\theta)}\left(R_s - \frac{\eta_{1s}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) - \frac{\eta_{3s}}{\eta_{3w}}R_w\right) + \frac{\eta_{1\theta}}{\eta_{1b}}\left(R_b - \frac{\eta_{3b}}{\eta_{3w}}R_w\right) + \frac{\eta_{3\theta}}{\eta_{3w}}R_w\right]}_{\gamma\eta_{2u'}} \\
= \quad & \beta R'_u + \beta\underbrace{\frac{1-\delta-f(\theta')s'}{u'f(\theta')}\left[R'_s - \frac{\eta'_{1s}}{\eta'_{1b}}\left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}}R'_w\right) - \frac{\eta'_{3s}}{\eta'_{3w}}R'_w\right]}_{-\lambda'\eta'_{0u}} \\
& -\beta\underbrace{\frac{\eta'_{1u}}{\eta'_{1b}}\left(R'_b - \frac{\eta'_{3b}}{\eta'_{3w}}R'_w\right)}_{\mu'\eta'_{1u}} - \beta\underbrace{\frac{\eta'_{3u}}{\eta'_{3w}}R'_w}_{\nu'\eta'_{3u}} \qquad \text{(B.7)}
\end{aligned}$$

where $\tilde{\eta}_{0,t}(u_t, s_t, \theta_t, u_{t+1}) := u_{t+1} - \delta(1-u_t) - (1 - f(\theta_t)s_t)u_t$

B.1.4 Equivalent Definition of Markov Perfect Equilibrium

This section provides an alternative and equivalent definition for the Markov perfect equilibrium where the government chooses b only and the private sector acts optimally. The derivation of Markov GEE for this definition is given after the definition.

Definition 6. (Markov Perfect Equilibrium) A Markov perfect equilibrium consists of a value function $\Omega(u)$, government's policy functions $\Psi(u)$ and $\tilde{\Gamma}(u,b)$, and private decision rules $\tilde{W}(u,b)$ $\tilde{S}(u,b)$, $\tilde{\Theta}(u,b)$ and $\tilde{\Pi}(u,b)$ solving

- for all u

$$\Psi(u) \in \arg\max_b R\left(u, b, \tilde{\Gamma}(u,b), \tilde{W}(u,b), \tilde{S}(u,b)\right) + \beta\Omega\left(\tilde{\Pi}(u,b)\right)$$

- for all u and b

$$\tilde{\Gamma}(u,b) = ub \qquad \text{(B.8)}$$

$$\tilde{\Pi}(u,b) = \delta(1-u) + \left[1 - f(\tilde{\Theta}(u,b))\tilde{S}(u,b)\right]u \qquad \text{(B.9)}$$

$$0 = \eta_1\left(u, b, \tilde{\Gamma}(u,b), \tilde{S}(u,b), \tilde{\Theta}(u,b), \tilde{\Pi}(u,b)\right) \qquad \text{(B.10)}$$

$$0 = \eta_2\left(\tilde{\Theta}(u,b), \tilde{\Pi}(u,b)\right) \qquad \text{(B.11)}$$

$$0 = \eta_3\left(u, b, \tilde{\Gamma}(u,b), \tilde{W}(u,b), \tilde{S}(u,b), \tilde{\Theta}(u,b)\right) \qquad \text{(B.12)}$$

- for all u

$$\Omega(u) \equiv R\left(u, \Psi(u), \tilde{\Gamma}(u,b), \tilde{W}(u,\Psi(u)), \tilde{S}(u,\Psi(u))\right) + \beta\Omega\left(\tilde{\Pi}(u,\Psi(u))\right)$$

i.e. the government moves first, choosing b and τ, then private sector moves according to (B.9)-(B.12). First-order condition of the government's problem, using (B.9) to substitute out τ and suppressing functional arguments

$$R_b + R_w\tilde{W}_b + R_s\tilde{S}_b + \beta\Omega'_u\tilde{\Pi}_b = 0 \tag{B.13}$$

differentiating Bellman equation with respect to u

$$\begin{aligned}\Omega_u &= R_u + R_b\Psi_u + R_w\left[\tilde{W}_u + \tilde{W}_b\Psi_u\right] + R_s\left[\tilde{S}_u + \tilde{S}_b\Psi_u\right] \\ &\quad +\beta\Omega'_u\left[\tilde{\Pi}_u + \tilde{\Pi}_b\Psi_u\right]\end{aligned} \tag{B.14}$$

substitute expression for $\beta\Omega'_u$ from (B.13) into (B.14)

$$\begin{aligned}\Omega_u &= R_u + R_b\Psi_u + R_w\left[\tilde{W}_u + \tilde{W}_b\Psi_u\right] + R_s\left[\tilde{S}_u + \tilde{S}_b\Psi_u\right] \\ &\quad -\frac{R_b + R_w\tilde{W}_b + R_s\tilde{S}_b}{\tilde{\Pi}_b}\left[\tilde{\Pi}_u + \tilde{\Pi}_b\Psi_u\right]\end{aligned}$$

update one period and substitute into (B.13)

$$\begin{aligned}0 &= R_b + R_w\tilde{W}_b + R_s\tilde{S}_b + \beta\tilde{\Pi}_b\Bigg\{R'_u + R'_b\Psi'_u + R'_w\left[\tilde{W}'_u + \tilde{W}'_b\Psi'_u\right] + R'_s\left[\tilde{S}'_u + \tilde{S}'_b\Psi'_u\right] \\ &\quad -\frac{R'_b + R'_w\tilde{W}'_b + R'_s\tilde{S}'_b}{\tilde{\Pi}'_b}\left[\tilde{\Pi}'_u + \tilde{\Pi}'_b\Psi'_u\right]\Bigg\}\end{aligned}$$

re-arrange to get the GEE

$$\begin{aligned}0 &= \left[R_b + \tilde{W}_bR_w + \tilde{S}_bR_s\right] + \beta\tilde{\Pi}_b\left[R'_u + \tilde{W}'_uR'_w + \tilde{S}'_uR'_s\right] \\ &\quad +\beta\tilde{\Pi}_b\left(-\frac{\tilde{\Pi}'_u}{\tilde{\Pi}'_b}\right)\left[R'_b + \tilde{W}'_bR'_w + \tilde{S}'_bR'_s\right]\end{aligned} \tag{B.15}$$

B.2 Sensitivity Analysis

For each value of ϕ, other parameters are recalibrated to match first-moments of the Markov equilibrium. The following table provides steady state values for the Ramsey and Markov economy. The steady states are not sensitive to changes to ϕ.

Table B.1: Sensitivity Analysis for Different ϕ Values in Steady State

	$\phi = 0.5$		$\phi = 1$ (baseline)		$\phi = 2$	
Statistic	Ramsey	Markov	Ramsey	Markov	Ramsey	Markov
benefit	0.314	0.564	0.318	0.564	0.325	0.565
wages	0.976	0.981	0.976	0.981	0.976	0.981
search	0.716	0.329	0.690	0.329	0.649	0.332
vacancy	0.036	0.028	0.036	0.028	0.036	0.028
unemployment	0.022	0.054	0.023	0.054	0.025	0.054

B.3 Additional Plots

B.3.1 Additional Policy Function Plots

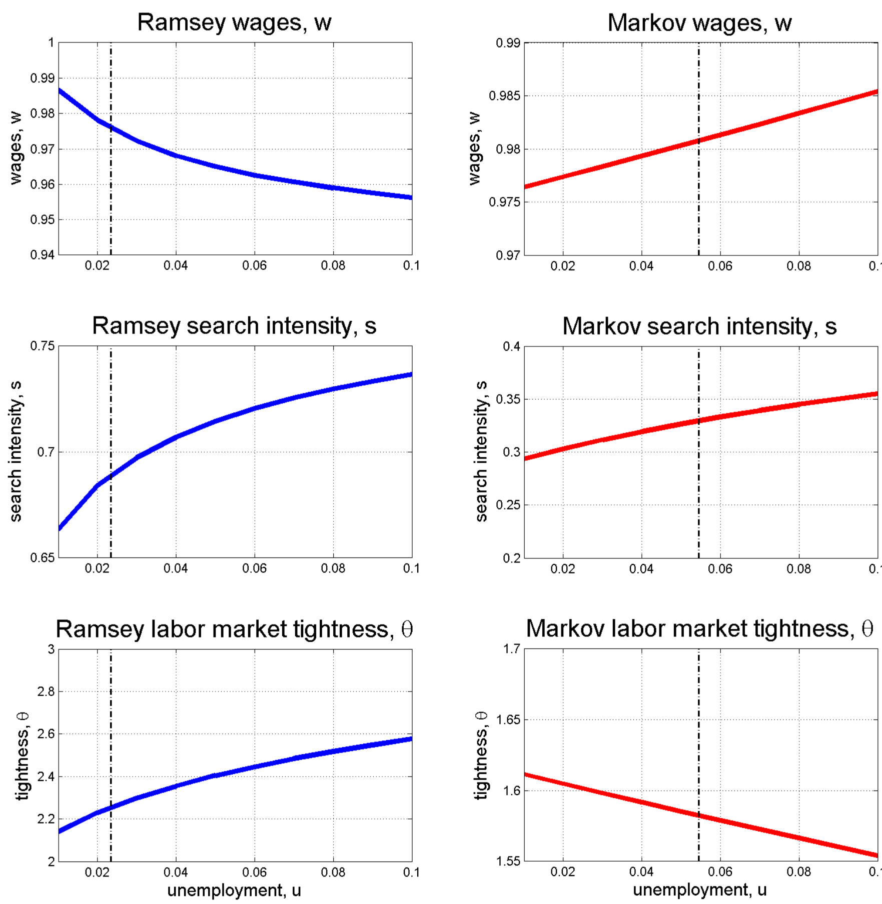

Figure B.1: Ramsey (left) and Markov (right) wage (top panel), search intensity (middle panel) and market tightness (bottom panel) policy functions holding productivity at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady state unemployment level.

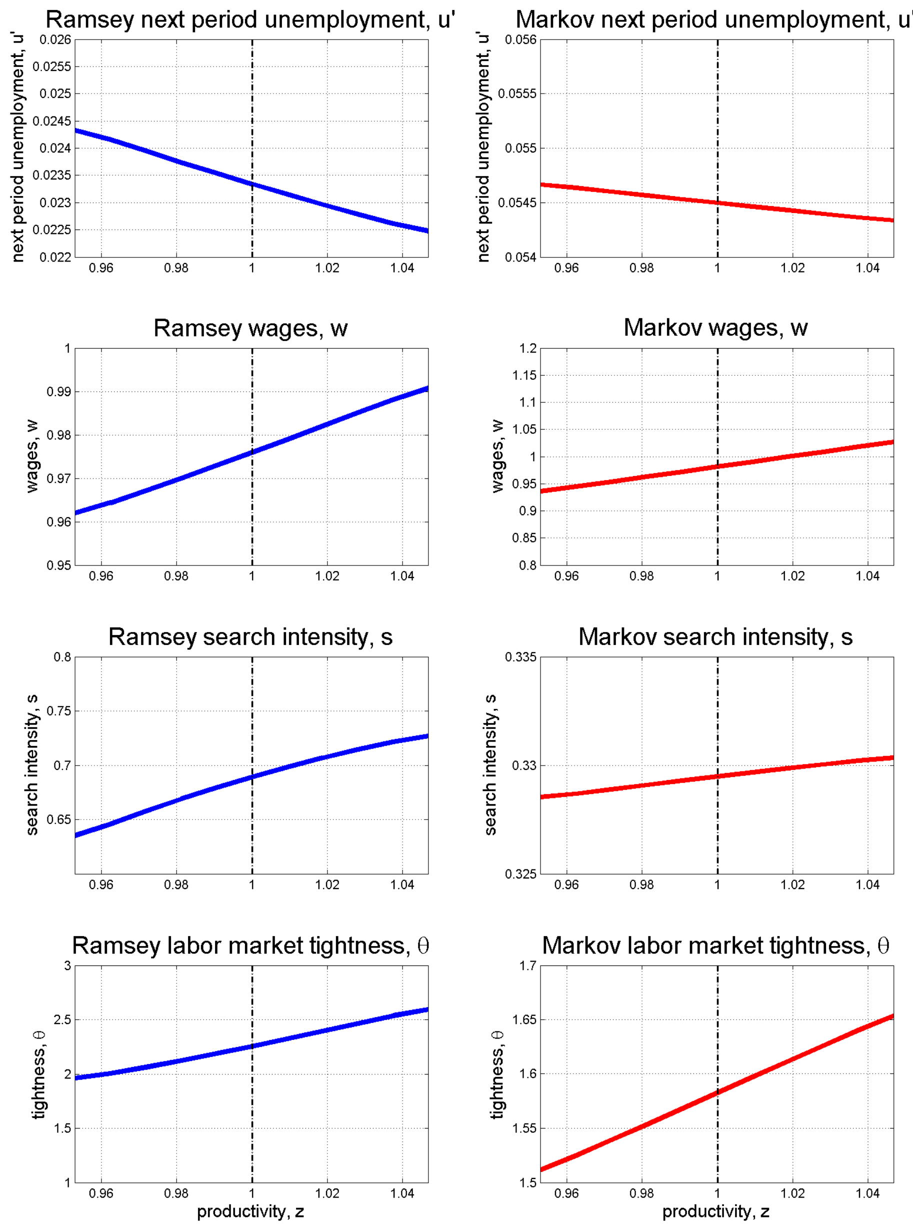

Figure B.2: Ramsey (left) and Markov (right) unemployment, wage, search intensity and market tightness policy functions holding unemployment at steady state. In each plot, the solid line denotes policy function, and the dashed line indicates steady state productivity level.

B.3.2 Additional Impulse Response Plots

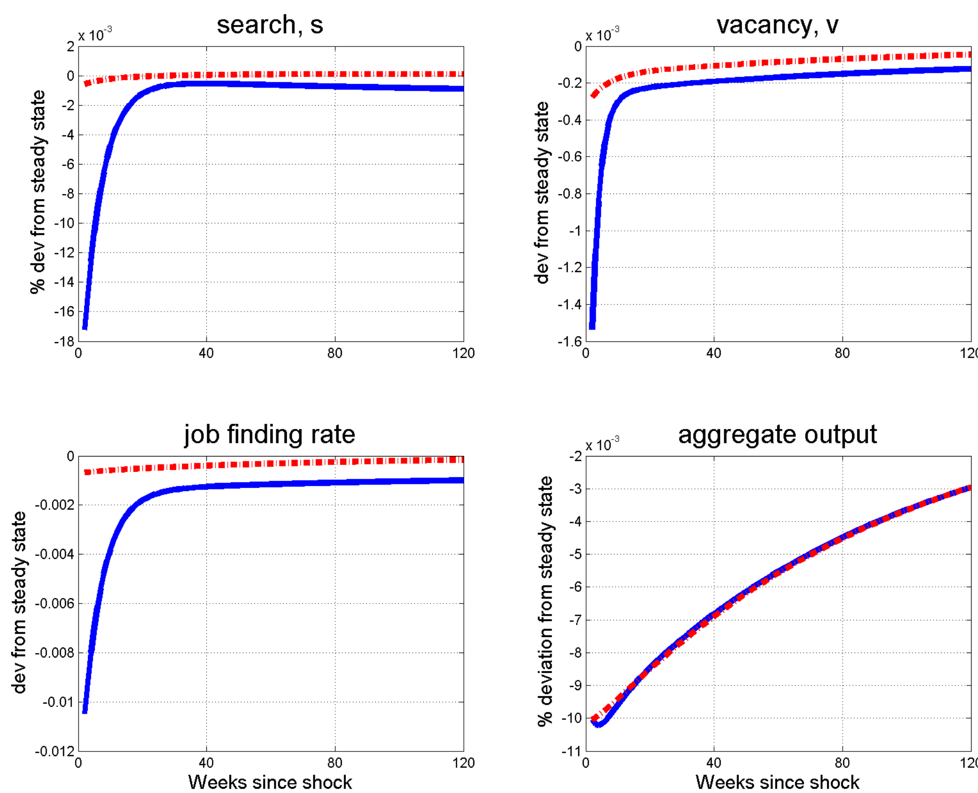

Figure B.3: Ramsey (solid line) and Markov (dashed line) responses to a 1% drop in productivity.